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Searching for Coherence between Spatial and Temporal Measures of Price Change

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Analytical Services Branch

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ABBREVIATIONS

CC	Collective consumption
DFD	Domestic final demand
ECP	European comparison program
GDP	Gross domestic product
GFCE	Gross fixed capital expenditure
IAC	Individual actual consumption
IPD	Implicit price deflator
PPP	Purchasing power parities

SEARCHING FOR COHERENCE BETWEEN SPATIAL AND TEMPORAL MEASURES OF PRICE CHANGE

Getachew A. Tessema and Peter D. Rossiter
Analytical Services

1. INTRODUCTION

Estimates of Purchasing Power Parity (PPP) provide the key to making meaningful comparisons of prices and incomes between countries. However, the rigorous data requirements and international coordination effort underlying their computation impose restrictions on the frequency of PPP benchmark programmes and the timeliness of published results. Consequently it has been common practice for international statistical agencies in the PPP programme to synthesise annual estimates by interpolating and projecting PPP benchmark data, using estimates of annual price change and real growth from the time-series system of national accounts of member countries.

While it is often observed that the time series-based projections of PPP benchmarks lead to inconsistent estimates, the reasons for the inconsistency, however, are generally not well understood. Conceptually, such inconsistencies are expected to arise between spatial and temporal indexes (Dalgaard and Sorensen, 2002). However, in practice it is imperative to understand the composition of the differences between the two estimates and to what extent these are explained by the various factors which may contribute to poor prediction outcomes such as data inconsistencies and measurement errors and those related to theoretical inconsistencies.

Acknowledging the inherent theoretical inconsistencies, Rhoades (2003) proposed a comprehensive mathematical description in which its aim is to decompose the discrepancy between the time series-based and PPP-implied measures of real growth in to its sources, namely,

- differences in basic input data to the time-series and PPP systems;
- index weighting patterns and adjustments;
- data revisions; and
- chaining.

In this paper we seek to gain a clearer understanding of Rhoades' methodology by taking the empirical approach using data provided by the Eurostat on OECD countries. Specifically, in our empirical case study we investigate aggregate temporal price (and volume) change in Ireland ('subject' country), using both the United

Kingdom and Germany as 'reference' countries. In particular, we examine price and expenditure data for a three-component disaggregation of *Domestic final demand – Individual actual consumption*, *Collective consumption* and *Gross fixed capital expenditure* – for the PPP benchmark years of 1999 and 2002.

Our analysis shows that, although Rhoades' decomposition of the discrepancy into price, expenditure and index formula substitution effects is not orthogonal or unique, our 'conditional' effects appear to have good practical implications for *post hoc* analysis. Accordingly, the impact of price differences between the time series-based and PPP systems play a dominant role in their relative contributions for the inconsistency between the PPP-implied and the time series-based measures of real price (and volume) growth. Our case study also discovers some unusually large 'theoretical' inconsistencies when index formulae are substituted. Interestingly, we discover that the relative contributions of the three sources of inconsistency (prices, expenditure and index formulae) vary as we change the reference country. These and other useful observations and recommendations are discussed in this paper.

Assuming that our results remain robust over time, our study also suggests a number of potentially useful areas in which Rhoades' methodology may be used in *post hoc* analysis to inform the calculation of future PPP projections.

Section 2 presents the methodology and describes the main features of Rhoades's (2003) analytical framework.

Section 3 describes the source of our data and the assumptions and modifications we have employed to construct a consistent and instructional database.

Section 4 presents preliminary analysis of the data by providing an empirical overview of the underlining inconsistencies between the time series-based and PPP-implied growth measures of real growth.

In Section 5, we step through Rhoades' decomposition methodology via a sequence of substitutions in which information from the PPP programme is used to replace the time series-based data. Following this (in Section 5.2) we present a detailed analysis and discussion of the main results from our case study.

In Section 6 we summarise our findings and conclude with some observations on the potential of Rhoades' methodology for computing enhanced PPP forecasts in real time.

2. METHODOLOGY

In this section we shall extract the key results from Rhoades' (2003) paper, an analytical framework employed in this study. After discussing the key features of the methodology we then highlight the main steps and procedures we have followed to empirically implement the analytical framework.

The consistency of spatial and temporal price and volume indexes is an issue of great importance for both users and producers of statistics. Almost all international comparisons of GDP or GDP per capita or their components are done by means of PPP converted data. However the theoretical literature on index numbers typically deals with either spatial or temporal indexes and rarely addresses the two indices simultaneously (see Dalgaard and Sorensen, 2002, p. 2).

While acknowledging the inherent theoretical inconsistencies, Rhoades (2003) attempts to heuristically quantify the coherence between spatial and temporal indexes by the concept of *PPP-implied real growth*. This measure seeks to predict real growth between PPP benchmark years in 'subject' country *S* from

- the change between benchmark years in the PPP-based output volume of the 'subject' country relative to the output volume of a 'reference' country *R*, and
- the time series-based growth estimate for the 'reference' country over the same period.

Comparison of the *PPP-implied real growth* estimate with the time series-based estimate of real growth for country S provides a test of the efficacy of using time series-based data to interpolate and project PPP benchmarks.

Rhoades' framework is based on the following key definition:

> "Intuitively, if the real output of country *S* is 10% that of country R in 1999 and 12% in 2002, then real growth in country S may be presumed to have been 20% higher than the real growth in country *R* between benchmark years."

As a way of introducing Rhoades' framework let us define:

$$GDP_t^S = \sum_i p_{it}^S q_{it}^S \quad \text{and} \quad GDP_t^R = \sum_i p_{it}^R q_{it}^R$$

as the national accounts based estimates of Gross Domestic Product (GDP) [1] for country *S* and *R* , respectively, at time *t*, *i* stands for the i^{th} expenditure category.

1 Note that while we use GDP-level of aggregation for illustrative purposes Rhoades' methodology is applicable at any level of aggregation.

Similarly, let the price ratios

$$\frac{p_{it}^S}{p_{it}^R} \text{ and } \frac{p_{it+k}^S}{p_{it+k}^R}$$

represent the binary Fisher estimates of PPP for the i^{th} expenditure category at time t and $t+k$. These PPPs can be calculated based on two sources; directly from the ECP program (ECP-based) or indirectly from the expenditure data of the time series-based System of National Accounts (SNA). For convenience, the respective ECP-based estimates will be denoted by the symbol '~' to distinguish them from the time series-based estimates.

At the GDP level the EKS procedure is applied to achieve transitivity across countries. That is, the PPP for country S relative to R is defined as:

$$PPP_t^{S(R)} = \left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} \left(\sum_i w_{it}^R \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} + \varepsilon_t^{S(R)} \qquad (1)$$

where $w_{it}^{S(R)}$ and w_{it}^R represent value shares (in R's currency) from the national accounts estimates for the i^{th} expenditure category for countries S and R, respectively, and are defined as

$$w_{it}^R = \frac{p_{it}^R q_{it}^R}{\sum_i p_{it}^R q_{it}^R}$$

and

$$w_{it}^{S(R)} = \frac{p_{it}^R q_{it}^S}{\sum_i p_{it}^R q_{it}^S}$$

$\varepsilon_t^{S(R)}$ represents the combined adjustment introduced by the EKS and linking at the GDP level.

Thus GDP for S in R's currency at time t is given by:

$$GDP_t^{S(R)} = \sum_i p_{it}^S q_{it}^S \left[\left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} \left(\sum_i w_{it}^R \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} + \varepsilon_t^{S(R)} \right]^{-1} \qquad (2)$$

After converting country S's GDP to R's currency, the real growth in S, as implied by PPP, can be expressed as the real growth in the reference country R multiplied by the growth in the GDP proportion of S relative to R. That is,

$$INT^S_{t+k} = \frac{\sum_i p^R_{it} q^R_{it+k}}{\sum_i p^R_{it} q^R_{it}} \frac{\dfrac{\sum_i p^S_{it+k} q^S_{it+k}}{\sum_i p^R_{it+k} q^R_{it+k}}}{\dfrac{\sum_i p^S_{it} q^S_{it}}{\sum_i p^R_{it} q^R_{it}}} \frac{\left[\left(\sum_i w^{S(R)}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i w^{R}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2} + \varepsilon^{S(R)}_{t+k}\right]^{-1}}{\left[\left(\sum_i w^{S(R)}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i w^{R}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2} + \varepsilon^{S(R)}_{t}\right]^{-1}} \quad (3)$$

Multiplying and dividing equation (3) by the real growth in S between periods t and $t+k$ and rearranging terms gives:

$$INT^S_{t+k} = \frac{\sum_i p^S_{it} q^S_{it+k}}{\sum_i p^S_{it} q^S_{it}} \frac{\dfrac{\sum_i p^S_{it+k} q^S_{it+k}}{\sum_i p^S_{it} q^S_{it+k}}}{\dfrac{\sum_i p^R_{it+k} q^R_{it+k}}{\sum_i p^R_{it} q^R_{it+k}}} \frac{\left[\left(\sum_i w^{S(R)}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i w^{R}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2} + \varepsilon^{S(R)}_{t+k}\right]^{-1}}{\left[\left(\sum_i w^{S(R)}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i w^{R}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2} + \varepsilon^{S(R)}_{t}\right]^{-1}} \quad (4)$$

where w^R_{it+k} and $w^{S(R)}_{it+k}$ are value shares at time $t+k$, as defined above in (1).

Rhoades (2003) refers to (4) as an intuitive definition or *PPP-implied real growth rate*. Thus the *PPP-implied real growth rate* in S is equal to the real growth rate in S, times the ratio of the implicit deflator in S relative to R, divided by the ratio of PPPs at time $t+k$ relative to time t.

> "Note that if the PPP is estimated by the ratio of implicit deflators, as is often done for the purpose of projecting PPPs between benchmarks, then INT_{t+k} will exactly equal real growth in S. This gives us some confidence that INT_{t+k} is a sensible concept despite the problems associated with its interpretation when actual benchmark PPPs are used in its calculation instead of deflator based projections." (Rhoades, 2003, p. 7).

The difference between the intuitive *PPP-implied* growth, INT_{t+k}, and the *time series based growth*, $\frac{\sum_i p^S_{it} q^S_{it+k}}{\sum_i p^S_{it} q^S_{it}}$, is embedded in the ratio of implicit deflators divided by the ratio of PPPs.

To simplify notation, equation (4) is rewritten as:

$$INT^S_{t+k} = \frac{\sum_i p^S_{it} q^S_{it+k}}{\sum_i p^S_{it} q^S_{it}} \times \Delta_{t+k}$$

where

$$\Delta_{t+k} = \frac{\dfrac{\sum_i p^S_{it+k} q^S_{it+k}}{\sum_i p^S_{it} q^S_{it+k}}}{\dfrac{\sum_i p^R_{it+k} q^R_{it+k}}{\sum_i p^R_{it} q^R_{it+k}}} \frac{\left[\left(\sum_i w^{S(R)}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i w^{R}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2} + \varepsilon^{S(R)}_{t+k}\right]^{-1}}{\left[\left(\sum_i w^{S(R)}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i w^{R}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2} + \varepsilon^{S(R)}_{t}\right]^{-1}} \quad (5)$$

Given this expression, a value of Δ_{t+k} equal to one implies no difference between the ECP-based (PPP-implied) and the time-series time series-based growth rates. On the other hand a value greater than one indicates the former is greater than the latter and *vice versa*. That is, the further it is from unity the larger the discrepancy between the spatial PPP-based estimates and time-series time series-based real growth rates. Thus empirically Δ_{t+k} can be used as measure of diagnostics for inconsistency between the two systems.

In order to assess the contributions of the various components further, Rhoades provides a series of mathematical re-expressions of equation (4) to decompose the effects of:

- differences in basic input data to the time series and PPP systems;
- index weighting patterns and adjustments;
- data revisions; and
- chaining.

Further details of these expressions are presented in Appendix A (see also Rhoades (2003) for further details and related assumptions).

It is important to note that, while innovative, some aspects of Rhoades' assumptions and empirical illustrations can be open to different interpretations. However, the essential elements of the methodology, as employed in our case study, can be summarised in simple terms as follows:

1. Compute the time series-based estimates of real growth.
2. Compute *PPP-implied* real growth (as defined by equation 4) and compare this with its corresponding estimates in step (1).
3. Sequentially substitute data items from the ECP data set (the data set that underlies the calculation of benchmark PPPs) for their corresponding time-series based data items. In each stage of substitution, re-compute a new estimate of *PPP-implied real growth* (using equation 4) and analyse the impacts of the particular substitution against the estimates in steps (1) and (2).
4. Examine the relative contributions of the various substitutions (step 3) and identify the key factors (or data items) that contribute to the inconsistency between *PPP-implied* (step 2) and the time-series based *estimates* (step 1) of real growth.

Finally, as illustrated in the above discussion, it is clear to note that the focus of Rhoades' analytical framework is on measures of real growth. However, we wish to explore the methodology further by defining the analogous concept of *PPP-implied price change* as a measure of aggregate price, rather than the derivative measure of real growth. This *PPP-implied price change* is derived by combining

- changes in the purchasing power parity of the 'subject' country relative to the 'reference' country, as measured in successive benchmark years, and
- the time series-based measure of implicit price change in the 'reference' country between benchmark years.

Specifically, we define *PPP-implied price change* as:

$$PPP\text{-}implied\ \ price\ change\ in\ S \quad = \frac{\sum_i p^R_{it+k} q^R_{it+k}}{\sum_i p^R_{it} q^R_{it+k}} \ \frac{\left(PPP^{S(R)}_{t+k} \Big/ PPP^{R}_{t+k} \right)}{\left(PPP^{S(R)}_{t} \Big/ PPP^{R}_{t} \right)} \qquad (6)$$

where $PPP^{S(R)}$ and PPP^{R} are the PPPs as defined in equation (1) for countries *S* and *R*, respectively, at time $t+k$ relative to time t, and $\frac{\sum_i p^R_{it+k} q^R_{it+k}}{\sum_i p^R_{it} q^R_{it+k}}$ is the time series-based implicit price change of R, at time $t+k$ relative to time t.

Thus, *PPP-implied price change* in S is equal to the time series-based implicit price change in R, times the ratio of PPPs in *S* relative to *R*, at time $t+k$ relative to time t. Note that Equation (6) is analogous to that of Rhoades's definition for *PPP-implied real growth* given by equation (3).

By comparing *PPP-implied price change* with change in the time series-based implicit price deflator for the subject country we can arrive at equivalent conclusions to the growth-based comparison described earlier.

3. DATA SOURCES AND ISSUES

3.1 Data sources

The primary data for our case study have been extracted from the 1999 and 2002 aggregate databases of the European Comparison Programme (ECP) – part of the Eurostat–OECD PPP Programme. We gratefully acknowledge the assistance of Eurostat in providing the relevant data.

Key data items in the ECP dataset include measures of international relative prices (PPPs), relative per capita output volumes and relative per capita expenditures, together with nominal expenditures, exchange rates and population estimates.

A second dataset, also kindly supplied by Eurostat, contains recent data from the national accounts of member countries. Key data items in the time series dataset include time-series of Gross Domestic Product and its components, measured in both current and constant prices. Appendix B provides a summary of the data.

Although considerable effort has been expended on ensuring consistency in the definition and scope of the respective ECP and time series expenditure aggregates, it must nonetheless be acknowledged that differences remain. At the national level, ECP and time series data (especially price data) have not necessarily been obtained from the same statistical collections, nor have they necessarily been compiled on the basis of identical statistical processes and economic assumptions. By focussing on highly aggregated data, we therefore accept the possibility that many of the inconsistencies we uncover in our analysis may simply arise from such mundane considerations.

If, in the future, we are able to obtain more detailed and consistent disaggregated data from both ECP and time series sources, we would expect to provide better insights into the current exercise.

3.2 Data issues

In our case study we have consciously chosen to analyse *Domestic final demand* (rather than *Gross domestic product*, for example) to minimise any data inconsistencies and avoid price issues surrounding aggregates such as *Change in inventories* and *Net imports* that might have some bearing on our moderate objective.

Differences between ECP and time series expenditure estimates should not necessarily be attributed to differences in scope or definition. Many differences are the consequence of revisions made to the time-series data. While some revisions may indeed result from new definitions or data sources, most arise from the time lags associated with processing the underlying source data. Price data may also be revised

over time, but most (aggregate) price revisions are the result of changes to the implicit expenditure weights.

In table 3.1 we summarise the nominal expenditures and purchasing power parity (PPP) estimates for *Individual actual consumption*, *Collective consumption* and *Gross fixed capital expenditure* extracted from the ECP dataset. German and Irish expenditures are denominated in Euros, and UK expenditures in Pounds Sterling.

In both benchmark years, we have defined German prices as the reference (index = 1.0). This re-referencing of PPPs to German prices (or the prices of any other country in the database) is necessary because the original PPPs have been calculated via the EKS methodology applied to the EU–15 Grouping of countries. In future discussion, however, we shall overlook the inherent EKS adjustments at the component level, and simply assume that all *component* PPPs provide accurate pairwise price comparisons between countries.

Expenditure on *Domestic final demand* is calculated as the sum of nominal expenditures on the three component sub-aggregates. The aggregate PPPs for *Domestic final demand* have been calculated, for the purposes of this study, by applying the EKS methodology to the PPPs of the three component sub-aggregates for the three selected countries. In later Sections, we shall discuss the EKS adjustment to *aggregate* PPPs as one of many factors influencing the comparison of PPP-implied and time series-based measures of temporal price change.

All volume estimates in table 3.1 have been calculated by dividing expenditures by their respective price measures.

Table 3.2 has been constructed from the time series accounts of Germany, Ireland and the United Kingdom, *circa* 2004. Current price expenditure estimates for *Individual actual consumption*, *Collective consumption* and *Gross fixed capital expenditure* are denominated in Euros and Pounds Sterling, consistently with the ECP estimates. Expenditure on *Domestic final demand* is again the sum of component expenditures.

The time series component price measures are implicit price deflators (IPDs), calculated as the ratio of published current price estimates to their corresponding constant price estimates, and assigned a common reference year of 1999=1.0. Re-referenced constant price, or volume, measures have then been calculated by dividing current price expenditures by the re-referenced IPDs.

In this re-referenced framework, the volume (constant price) measure of *Domestic final demand* may be calculated as the sum of the component volume measures, and the aggregate IPD may then be derived as the ratio of total expenditure to total volume. Equivalently, the total volume for 2002 may be calculated as a Laspeyres index from the component volumes, and the aggregate IPD for 2002 may be calculated as a Paasche index from the component data.

3.1 Spatial benchmark data extracted from the databases of the European Comparisons Project – 1999 and 2002

	Individual actual consumption			*Collective consumption*			*Gross fixed capital expenditure*			*Domestic final demand*		
	Price	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*
1999												
Germany	1	1,375,800	1,375,800	1	158,340	158,340	1	426,390	426,390	1	1,960,530	1,960,530
Ireland	0.9493	53,669	50,949	0.7999	6,003	4,802	0.8609	24,869	21,409	0.9158	84,256	77,160
United Kingdom	0.6529	1,059,532	691,812	0.5438	123,266	67,034	0.6471	238,971	154,647	0.6424	1,422,039	913,493
2002												
Germany	1	1,479,550	1,479,550	1	168,100	168,100	1	391,760	391,760	1	2,039,410	2,039,410
Ireland	1.0835	64,444	69,826	0.9353	7,361	6,885	1.0172	28,472	28,963	1.0549	100,174	105,674
United Kingdom	0.6307	1,298,972	819,286	0.5658	144,628	81,835	0.6923	247,989	171,695	0.6365	1,685,536	1,072,816

Note: Expenditures/volumes are in '000 units

3.2 Temporal economic data collected from the time series databases of Germany, Ireland and the United Kingdom

	Individual actual consumption			*Collective consumption*			*Gross fixed capital expenditure*			*Domestic final demand*		
	Price	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*
Germany												
1999	1	1,394,840	1,394,840	1	167,410	167,410	1	428,420	428420	1	1,990,670	1,990,670
2002	1.0383	1,446,348	1,501,720	1.0212	169,941	173,540	0.9839	399,382	392970	1.0261	2,015,670	2,068,230
Ireland												
1999	1	51,047	51,047	1	4,827	4,827	1	21,663	21663	1	77,537	77,537
2002	1.1568	61,713	71,391	1.2140	5,716	6,940	1.2058	24,082	29039	1.1733	91,512	107,370
United Kingdom												
1999	1	692,610	692,610	1	66,778	66,778	1	156,344	156,344	1	915,732	915,732
2002	1.0687	768,537	821,346	1.1022	74,074	81,648	1.0162	170,754	173,525	1.0623	1,013,365	1,076,519

Note: Expenditures/volumes are in '000 units

4. PRELIMINARY ANALYSIS OF THE RESULTS

In this section, time series-based and PPP-implied aggregate price and volume measures of real growth are computed. We examine the apparent inconsistencies between the two sets of estimates. The impact of varying the reference country for PPP-implied measures is illustrated. We note the relative importance of price and expenditure inconsistencies at the component level and record apparent patterns in our results.

(a) Aggregate price and volume changes

Table 4.1 summarises the aggregate price movements which form the primary focus of our investigation.

4.1 Measures of temporal price change and real growth – Domestic final demand, 1999–2002

	Germany	*Ireland*	*United Kingdom*
Temporal price change			
Time series-based	2.61%	17.33%	6.23%
PPP-implied (Reference=Germany)	2.61% *	18.20%	1.67%
PPP-implied (Reference=Ireland)	1.85%	17.33% *	0.92%
PPP-implied (Reference=United Kingdom)	7.22%	23.51%	6.23% *
Real growth			
Time series-based	1.26%	18.02%	10.66%
PPP-implied (Reference=Germany)	1.26% *	15.73%	15.38%
PPP-implied (Reference=Ireland)	3.26%	18.02% *	17.66%
PPP-implied (Reference=United Kingdom)	–2.88%	11.00%	10.66% *

The time series-based measures of aggregate price change for *Domestic final demand*, between 1999 and 2002, can be read directly from the aggregate implicit price deflators in table 3.2.

The corresponding PPP-implied measures combine ratios of PPPs from table 3.1 with the time series-based price change in the reference country from table 3.2.[2] For completeness, when the subject and reference countries are the same, we define PPP-implied price change to be identical to the time series-based measure. These cases are identified by asterisks in table 4.1.

Note that the PPP-implied measures of aggregate price change differ quite significantly from one another (up to 5–6 percentage points), and from the time series-based

2 For example, PPP-implied price change for Ireland, using the United Kingdom as the reference country is

$$\left(\frac{1.0549/0.6365}{0.9158/0.6424} \times 1.0623 - 1\right) \times 100\% \quad = \quad 23.51\%.$$

measures. For example, PPP-implied price change in Ireland with Germany as reference country is 18.20% compared to 23.51% when the United Kingdom is used as reference country, implying a difference of 5.31 percentage points.

Intuitively, if we assume Germany and the United Kingdom to exhibit relatively stable and consistent economic behaviour over time, then we might expect smaller discrepancies between the PPP-implied and time series-based results for Ireland. However, observing large discrepancies is not a very encouraging result.

Note that there are two important characteristics of the results in table 4.1. A closer look at the PPP-implied price change in the three countries reveal that there is a fixed relationship in relative growth ratios which is not distorted by the choice of reference country. That is, the relative growth ratios are reference country invariant. For example, the relative growth ratio pertaining to aggregate price change for Germany is 86.81% (i.e. 1.261/1.1820 = 1.185/1.1733 = 1.0722/1.2351= 0.8681) of the growth ratio for Ireland. The corresponding value for the United Kingdom is 86.01%. On the other hand, the actual growth ratios are reference country dependent (see Appendix C for further discussion).

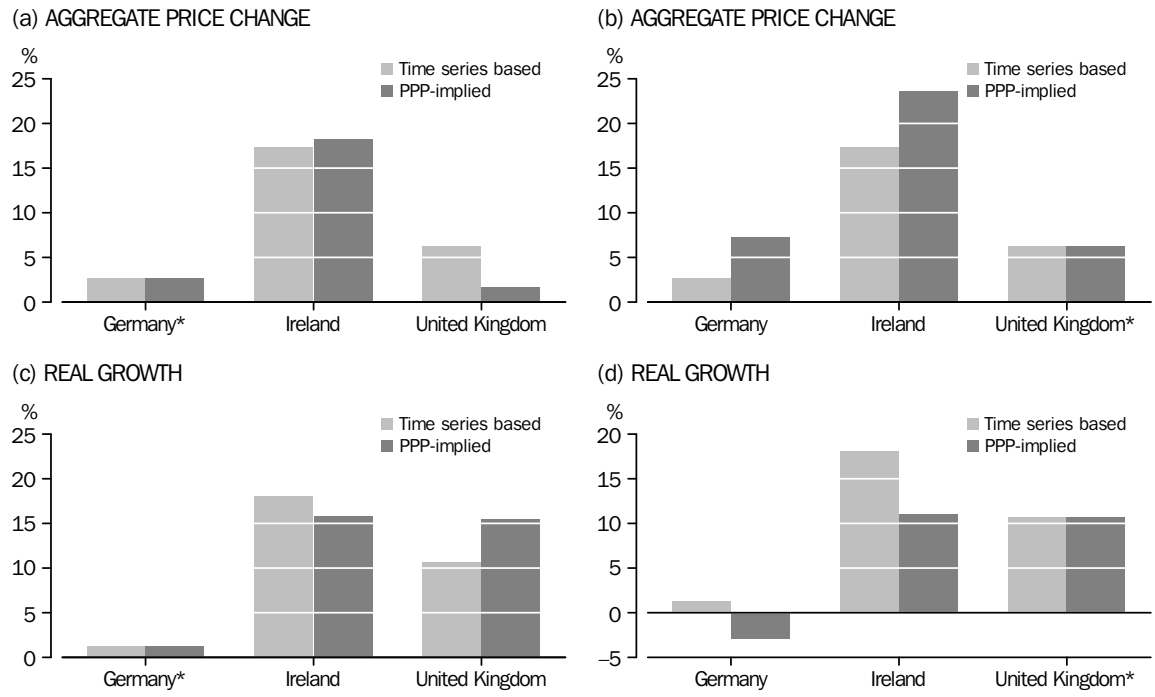
Thus, given that actual growth ratios are dependent on the choice of a reference country, it is logical to assume that either the United Kingdom or Germany will be 'closer' than the other to our chosen subject country of Ireland. In this case PPP-implied and time series-based price changes for Ireland based upon Germany as the reference country were found to be relatively closer to each other than estimates based upon the United Kingdom (see Appendix C).

Hence Germany represents the better choice of reference country for Ireland based upon this *post hoc* analysis of the data. This fact could, of course, have been deduced trivially from a comparison of figures 4.2(a) and 4.2(b).

The impacts of a choice of a reference country described for PPP-implied aggregate price change also apply to PPP-implied real growth. That is, growth ratio estimates of PPP-implied real growth based upon Germany as the reference country are closer to the time series-based Irish measures than the PPP-implied real growth referenced to the United Kingdom. Again, implying Germany as the better choice of reference country for Ireland – see also figures 4.2(c) and 4.2(d).

As expenditure revisions may impact asymmetrically upon price and volume measures, we must acknowledge that the choice of preferred reference country may, in some cases, differ depending upon whether we examine PPP-implied measures of price change or real growth. In the absence of expenditure revisions, the two approaches will yield identical conclusions. Where revisions are small (as in the present case), the probability of inconsistent conclusions is also small, and the implications of lesser consequence.

4.2 Time series-based and PPP-implied measures of aggregate price change and real growth – Domestic final demand, 1999–2002



* denotes the reference country for PPP-implied measures.

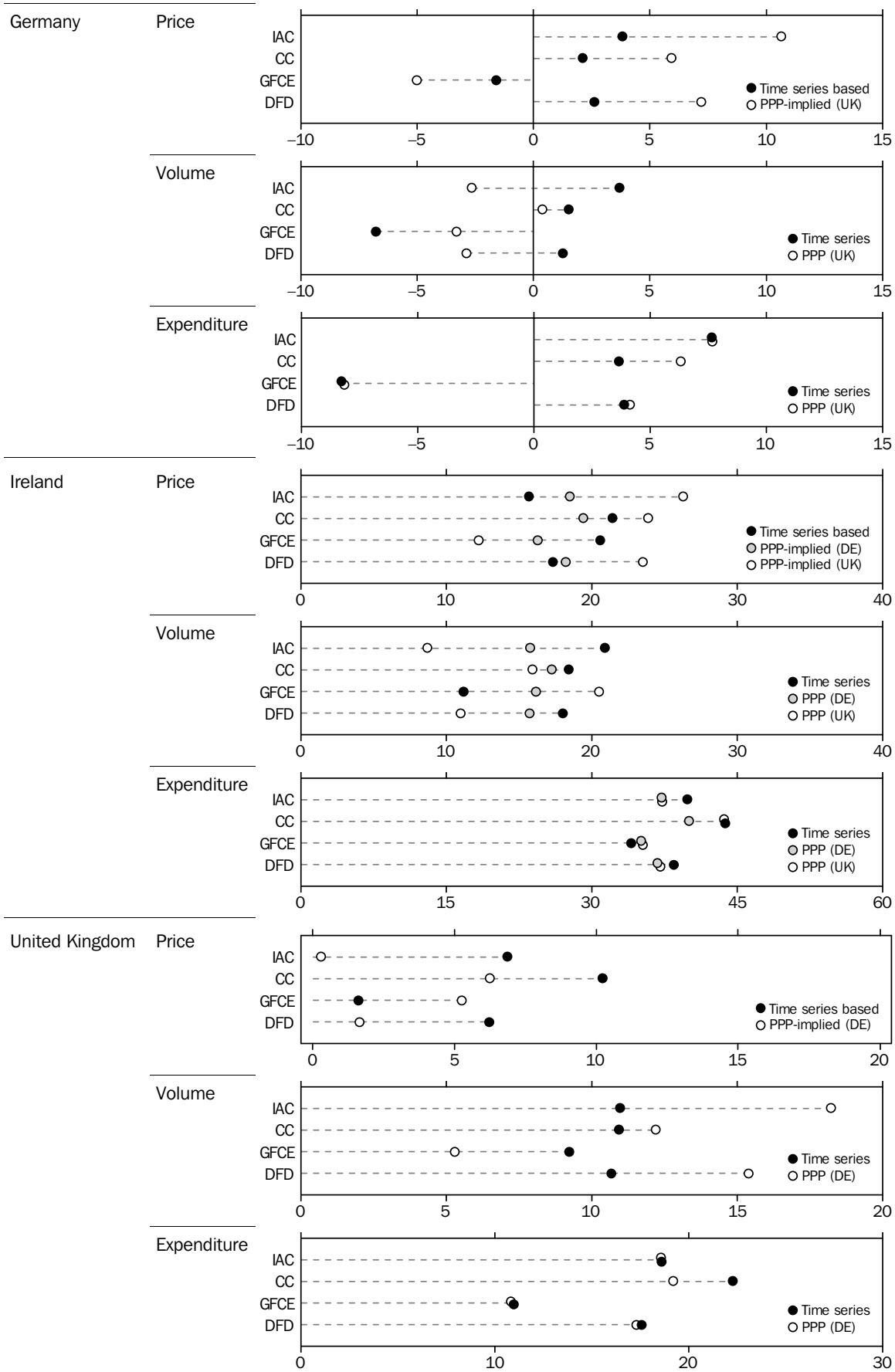
Intuitively, we expect to find that revisions to expenditure contribute a direct proportional effect to PPP-implied real growth measures, whereas PPP-implied aggregate price change is affected predominantly by changes in the *composition* of aggregate expenditure. We consider changes in composition to be inherently more informative about the differences between the PPP and time series collections than revisions to the level of expenditure. Hence we suggest that the choice of reference country is perhaps better informed by the PPP-implied price change measures.

(b) Sub-aggregate price and volume changes

In figure 4.3 we contrast time series-based and PPP-implied measures at the sub-aggregate level, using both Germany (DE) and the United Kingdom (UK) as reference countries as applicable.

Over all, there is a good consistency between time series-based and PPP-implied measures of total expenditure in *Domestic final Demand* for the three countries regardless of the choice of reference country. This is indicative of small revisions to total expenditure in all three countries. This is also true of the larger aggregates, *Individual actual consumption* (IAC) and *Gross fixed capital expenditure* (GFCE), but not of *Collective consumption* (CC).

4.3 Comparison of time series-based and PPP-implied growth measures – Domestic final demand and sub-aggregates, 1999–2002



That is, while time series-based and PPP-implied measures of total expenditure for *Individual actual consumption* (IAC) and *Gross fixed capital expenditure* (GFCE) show good consistency, this, however, is not true for *Collective consumption* (CC).

The source of this discrepancy can be traced back to tables 3.1 and 3.2 which show significant discrepancies in both levels and movements when the time series and PPP expenditure estimates of *Collective consumption* are compared. In Germany there appears strong evidence of inconsistent scope between the two collections, as PPP expenditures in *Collective consumption* are significantly lower than the time series estimates.

Figure 4.3 shows that the large discrepancy associated with PPP-implied price change for IAC in Germany, with the United Kingdom as reference country, is also mirrored in the PPP-implied real growth measure for IAC. These two discrepancies are, in turn, mirrored in the discrepancies in the corresponding UK measures which use Germany as a reference. Similar patterns of large inconsistencies are observed for GFCE measures.

Given these large discrepancies, we may conclude that Germany and the United Kingdom represent very poor choices of reference country for one another. That is, virtually all PPP-implied price and volume measures derived from this pairing of countries differ substantially from their time series-based counterparts. On the other hand, we should not overlook the possibility that the widespread discrepancies observed may result from perhaps only a single source of error or inconsistency – diffused through the various computations.

From the aggregate growth measures in figure 4.2, in comparing time series-based aggregate price change and real growth against their PPP-implied counterparts in Ireland, we have already established that PPP-implied measures referenced to Germany compare more favourably than PPP-implied measures referenced to the United Kingdom. From figure 4.3 we see that German-based PPP-implied measures are also closer than their UK-based counterparts at the sub-aggregate level.

However, this does not necessarily imply that there is always a particular reference country that is consistently preferable for the three sub-aggregates. In fact, it is possible that, time series-based and PPP-implied measures may be similar for one component but not another, and hence a particular country may be the best choice of reference country for one component and not for another component. We suggest only that if a particular country is found to be the best choice of reference country at the aggregate level (for example, *Domestic final demand*) then it is likely to also be the best choice for most of the sub-aggregates.

Despite Germany being a preferable reference country for Ireland, the time series-based and PPP-implied measures for Ireland based upon Germany still show

significant discrepancies between them. Perhaps given the high rates of both price inflation and real growth reported by Ireland between 1999 and 2002, the discrepancies we observe in figure 4.3 between the time series-based and PPP-implied measures for IAC and CC (based upon Germany) are not unduly worrisome. However, the time series-based and PPP-implied measures for GFCE are totally incompatible (as is also evident from table 4.3 of Appendix C).

The time series-based growth measures for Ireland in figure 4.3 indicate that the price of capital goods in Ireland appears to have risen much faster than the price of consumption goods, and consequently real growth in GFCE lags considerably behind real growth in consumption. By contrast, the time series data for Germany and the United Kingdom show growth in their GFCE price deflators to be much lower than growth in their corresponding consumption deflators. In fact the German GFCE deflator suggests that the price of capital fell between 1999 and 2002. The PPP-implied measures for Ireland (based upon both reference countries) also support this converse scenario, reporting very low growth in the price of capital.

We suggest that these conflicting scenarios may be attributed to

- significant differences between the mix of capital goods purchased in Ireland and the capital goods purchased in Germany and the United Kingdom, and/or
- significant differences in the statistical methods employed to value capital goods in the three countries. For example, Germany and the United Kingdom use hedonics to produce the deflators for computers in their investment series but not in Ireland.

In subsequent sections, we shall demonstrate how the observed inconsistencies between the PPP and time series data on component prices and expenditures influence the discrepancy between PPP-implied and time series-based measures of aggregate price change and real growth.

5. RESULTS FROM DECOMPOSITION

In the preceding section, we observed numerous differences between

(i) the expenditure estimates reported by the time series and ECP collections, and

(ii) time series-based and PPP-implied measures of price (and volume) change

at the aggregate as well as sub-aggregate level. While some differences appear to be the result of minor revisions, others are suggestive of inconsistent scope or measurement practices.

Earlier (in Section 3) we described the choices we made in combining data on the three component sub-aggregates to produce estimates of total expenditures, aggregate prices and real volumes for *Domestic final demand*. These aggregation methods, chosen to align with common practice in the production of temporal and spatial economic statistics, will ensure that time series-based and PPP-implied measures of growth are not identical – even in the unlikely event that identical price and expenditure data are collected by the time series and ECP statisticians. In later discussion, we shall refer to the distortions introduced by our choice of aggregation methods as 'theoretical' inconsistencies.

In this and the following section, we decompose the differences between time series-based and PPP-implied real growth and aggregate price change by examining the contributions of the three identified sources of inconsistency: component expenditures, component prices and 'theoretical' issues. While we may think of theoretical inconsistencies as those remaining after all other sources of inconsistency have been accounted for, we find it harder to disentangle the contributions of price and expenditure inconsistencies. This is because both prices and expenditures impact upon the measures of *real volumes* – which play an integral role in the aggregation process.

Below, we step through Rhoades' decomposition methodology via a sequence of substitutions in which information from the PPP programme is used to replace time series-based data. We are careful to explicitly describe each step in detail.

After describing the process in Section 5.1, we assess the results quantitatively in Section 5.2. We also look at the effects of changing the order of the substitutions, with a view to isolating the specific contributions of individual prices and expenditures (Rhoades (2003), p. 15). Empirically we will examine the relative contributions of the three sources of inconsistency as we change the reference country.

5.1 Formatting data for decomposition

Following Rhoades' decomposition methodology, we seek to reconcile the differences between time series-based and PPP-implied growth measures by means of a sequence of substitutions in which time series-based data are progressively replaced by data from the ECP collection. Firstly we reproduce the time series calculations after revising all component expenditures with their ECP-sourced counterparts. Then we replace the time series-based implicit price deflators at the component level with their PPP-implied equivalents, and finally we review the aggregation methodology.

We use table 5.1(a)–(d) to illustrate our understanding of Rhoades' methodology for reconciling time series-based and PPP-implied growth measures. The substitutions made in each step of the procedure are highlighted in bold and careful explanations are provided for each substitution underneath each table.

(a) The starting point – time series-based data

As a starting point, we require data which exhibit the time series-based measures of change in prices, volumes and expenditures. Of course table 3.2 is the most obvious source of such data. However, we prefer to present the data from table 3.2 in a slightly different format, the details of which are presented underneath table 5.1(a).

The outcome of all this manipulation is that temporal growth estimates derived from table 5.1(a) for prices, volumes and expenditures, for the three sub-aggregates and *Domestic final demand*, will be identical to the corresponding (time series-based) estimates derived from table 3.2.

The advantage of table 5.1(a) is that it brings together data from the time series and ECP systems in a common real-time scenario: time series data have been published for 1999 and 2002 and users are seeking to predict the outcome of the 2002 ECP benchmark programme by extrapolating the 1999 ECP benchmark data.

In effect, table 5.1(a) illustrates the case where the time series-based aggregate growth estimates are used to extrapolate 1999 benchmark data at the aggregate level (i.e. *Domestic final demand*). Viewing table 5.1(a) as extrapolated ECP-based data, we can show trivially that the derived PPP-implied temporal growth measures will be identical to the time series-based estimates of table 3.2. [3]

3 For example, in the case of aggregate prices for Ireland, with Germany as the reference country,

$$\frac{1.0745 \,/\, 1.0261}{0.9158 \,/\, 1} \times 1.0261 = \frac{1.0745}{0.9158} = 1.1733,$$

which is identical to the implicit price deflator from table 3.2.

5.1 Reconciliation of differences between time series-based and PPP-implied temporal growth measures

(a) The starting point – time series-based data

	Individual actual consumption			*Collective consumption*			*Gross fixed capital expenditure*			*Domestic final demand*		
	Price	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*
Germany												
1999	1	1,394,840	1,394,840	1	167,410	167,410	1	428,420	428,420	1	1,990,670	1,990,670
2002	1.0383	1,446,348	1,501,720	1.0212	169,941	173,540	0.9839	399,382	392,970	1.0261	2,015,670	2,068,230
Ireland												
1999	0.9493	53,773	51,047	0.7999	6,034	4,827	0.8609	25,164	21,663	0.9158	84,668	77,537
2002	1.0982	65,008	71,391	0.9711	7,147	6,940	1.0380	27,975	29,039	1.0745	99,928	107,370
United Kingdom												
1999	0.6529	1,060,754	692,610	0.5438	122,795	66,778	0.6471	241,594	156,344	0.6424	1,425,525	915,732
2002	0.6978	1,177,038	821,346	0.5994	136,212	81,648	0.6576	263,861	173,525	0.6824	1,577,510	1,076,519

Table 5.1(a) has been compiled as follows:

- We retain the published time series expenditure levels (in national currencies) for 1999 and 2002;
- The prices in 1999 (the base year for our constant price estimates) have been replaced by the 1999 benchmark purchasing power parities (referenced to Germany) from table 3.1;
- Volume measures for 1999 have been calculated by dividing expenditures by prices;
- Prices for 2002, at the sub-aggregate level, have been derived by extrapolating 1999 prices by growth in the corresponding time series deflators.
- Volume measures for 2002 have been computed by dividing 2002 expenditures by the projected prices;
- Aggregate price change (1999–2002) for *Domestic final demand* has been calculated as a Paasche index from the sub-aggregate price and volume estimates;
- Real growth in *Domestic final demand* can be calculated as a Laspeyres index from the sub-aggregate price and volume data. Alternatively, the volume measure for 2002 may be computed as expenditure divided by price – but not as the sum of component volumes.

5.1(b) Substitution of ECP expenditure estimates for time series expenditure estimates

	Individual actual consumption			*Collective consumption*			*Gross fixed capital expenditure*			*Domestic final demand*		
	Price	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*
Germany												
1999	1	1,375,800	**1,375,800**	1	158,340	**158,340**	1	426,390	**426,390**	1	1,960,530	1,960,530
2002	1.0383	1,424,995	**1,479,550**	1.0212	164,614	**168,100**	0.9839	398,152	**391,760**	1.0260	1,987,761	2,039,410
Ireland												
1999	0.9493	53,669	**50,949**	0.7999	6,003	**4,802**	0.8609	24,869	**21,409**	0.9158	84,256	77,160
2002	1.0982	63,583	**69,826**	0.9711	7,090	**6,885**	1.0380	27,901	**28,963**	1.0747	98,333	105,674
United Kingdom												
1999	0.6529	1,059,532	**691,812**	0.5438	123,266	**67,034**	0.6471	238,971	**154,647**	0.6424	1,422,039	913,493
2002	0.6978	1,174,086	**819,286**	0.5994	136,524	**81,835**	0.6576	261,078	**171,695**	0.6825	1,571,971	1,072,816

Table 5.1(b) has been compiled as follows:

- ECP expenditure estimates have been substituted for time series expenditures at the sub-aggregate level, and expenditure on *Domestic final demand* has been calculated as the sum of component expenditures;
- Price data for the three sub-aggregates have been retained from table 5.1(a);
- Volume measures at the sub-aggregate level have been recalculated by dividing the revised expenditure estimates by the former prices;
- Measures of aggregate price change and real growth (1999–2002) in *Domestic final demand* have been recalculated as previously, using the Paasche and Laspeyres index formulae, but with the revised volume measures.

For this aggregate extrapolation approach to provide a reasonable projection of the 2002 ECP benchmarks, we must trust that the time series-based extrapolations at the sub-aggregate level are accurate, and that theoretical inconsistencies in the aggregation process are small. The validity of these assumptions will be assessed later in this case study.

Comparison of tables 5.1(a) and 5.1(b) reveals sizeable revisions to *Collective consumption* and *Individual actual consumption* estimates in Germany and to *Collective consumption* in Ireland. For example, the time series-based expenditure on Collective consumption for Germany changed from 158,340 in the ECP-based data to 167,410 in the time series-based data for 1999, which represents a 5.7% change as a result of revisions. The corresponding change due to revisions for 2002 was about 3.2% (i.e. from 168,100 to 173, 840). Such revisions flow through directly to their corresponding volume measures (as component prices are unchanged) and thereby alter the estimates of real growth in *Domestic final demand*.

Looking at tables 5.1(a) and 5.1(b), the revisions to aggregate (*Domestic final demand*) price change are minor, not only because the component prices remained unchanged, but also because the revisions to real volumes were not large enough to significantly alter the volume weights in the Paasche index formula.

(c) Substitution of PPP-implied price changes for time series-based measures of price change at the sub-aggregate level

In tables 5.1(a) and 5.1(b), 1999 prices are the 1999 benchmark PPPs (referenced to Germany) and 2002 prices have been calculated by extrapolating the 1999 prices by time series-based implicit price deflators. We now wish to replace these time series-based implicit price movements with PPP-implied price movements, referenced to Germany. This is achieved by replacing 2002 prices in table 5.1(b) with the product of (a) the 2002 PPP benchmarks from table 3.1 and (b) the corresponding time series-based measures of price change in Germany. [4]

Comparing table 5.1(c) with table 5.1(b), we note that the implicit measure of growth in aggregate prices has risen slightly for Ireland and fallen for the United Kingdom, and consequently real growth is now lower in Ireland and higher in the United Kingdom.

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4 For example, for *Individual actual consumption* in Ireland, this calculation is

$$1.0835 \times 1.0383 = 1.1250$$

and the corresponding measure of PPP-implied price change is calculated as

$$\left(\frac{1.1250 \,/\, 1.0383}{0.9493 \,/\, 1} \times 1.0383 - 1 \right) \times 100\% = 18.51\%$$

5.1(c) Substitution of PPP-implied price changes for time series-based measures of price change at the sub-aggregate level

	Individual actual consumption			*Collective consumption*			*Gross fixed capital expenditure*			*Domestic final demand*		
	Price	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*
Germany												
1999	1	1,375,800	1,375,800	1	158,340	158,340	1	426,390	426,390	1	1,960,530	1,960,530
2002	1.0383	1,424,995	1,479,550	1.0212	164,614	168,100	0.9839	398,152	391,760	1.0260	1,987,761	2,039,410
Ireland												
1999	0.9493	53,669	50,949	0.7999	6,003	4,802	0.8609	24,869	21,409	0.9158	84,256	77,160
2002	**1.1250**	62,067	69,826	**0.9551**	7,209	6,885	**1.0009**	28,936	28,963	1.0801	97,838	105,674
United Kingdom												
1999	0.6529	1,059,532	691,812	0.5438	123,266	67,034	0.6471	238,971	154,647	0.6424	1,422,039	913,493
2002	**0.6549**	1,251,075	819,286	**0.5778**	141,629	81,835	**0.6812**	252,036	171,695	0.6520	1,645,437	1,072,816

Table 5.1(c) has been compiled in the following way:

- All ECP-based expenditure estimates from table 5.1(b) have been retained;
- All sub-aggregate prices for 2002 have been calculated via the method described above, effectively replacing time series-based implicit price changes with PPP-implied price changes (referenced to Germany);
- Volume measures at the sub-aggregate level have been recalculated by dividing the expenditure estimates by the revised prices;
- Measures of aggregate price change and real growth (1999–2002) in *Domestic final demand* have been recalculated as in table 5.1(b), using the Paasche and Laspeyres index formulae, but with the revised prices and volumes.

In Ireland, aggregate price change increases because the upward revision to the price of the dominant consumption component is sufficient to counter the large discrepancy between the time series-based and PPP-implied price measures for *Gross fixed capital expenditure* (which we noted earlier in our discussion of figure 4.3). For the United Kingdom, the major discrepancies occur in the price of consumption. However it is perhaps fair to observe that all price measures in table 5.1(c) have undergone significant change. Corresponding reciprocal movements may also be observed in the volume measures (since expenditures have been held constant).

Thus as both prices and volumes have changed significantly, it is likely that both have a significant impact on the Laspeyres and Paasche indexes used in aggregation.

The extent of revision to the component price measures may have arisen because of a difference in scope between the goods and services priced by the ECP and time series statisticians, or because of different valuation methods employed in the two systems. For instance, while Germany uses hedonics to produce the deflators for computers in its investment series Ireland does not.

(d) Substitution of Fisher index formulae for Paasche and Laspeyres formulae

As the 2002 indexes are already based upon component price and expenditure data from the ECP collection, it only remains to quantify the discrepancy between the 2002 price indexes in table 5.1(c) and the benchmark spatial price indexes for 2002, which are calculated as EKS price indexes from the same data.

In fact, we shall quantify this discrepancy in two stages. In the first stage, we make use of the Fisher index formula to create bilateral spatial price comparisons between subject country and reference country. We then attribute any remaining discrepancy to the EKS adjustments in 1999 and 2002.

In table 5.1(d), we have highlighted in bold the calculated Fisher spatial price indexes for Ireland and the United Kingdom, with Germany chosen as the reference country in both cases.

Comparing tables 5.1(d) and 5.1(c), we observe very small differences between the 1999 Fisher and EKS price indexes for *Domestic final demand* in both countries. In 2002, the Fisher price index for the United Kingdom is also very close to the corresponding Paasche index. In Ireland, by contrast, the Fisher and Paasche indexes for 2002 are noticeably different.

Volume indexes for *Domestic final demand* in 2002 are also calculated as Fisher indexes from the component data, although the identical result may be obtained by dividing the Fisher price index into total expenditure.

5.1(d) Substitution of Fisher index formulae for the Paasche and Laspeyres formulae

	Individual actual consumption			*Collective consumption*			*Gross fixed capital expenditure*			*Domestic final demand*		
	Price	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*	*Price*	*Volume*	*Expenditure*
Germany												
1999	1	1,375,800	1,375,800	1	158,340	158,340	1	426,390	426,390	1	1,960,530	1,960,530
2002	1.0383	1,424,995	1,479,550	1.0212	164,614	168,100	0.9839	398,152	391,760	1.0260	1,987,761	2,039,410
Ireland												
1999	0.9493	53,669	50,949	0.7999	6,003	4,802	0.8609	24,869	21,409	**0.9153**	84,296	77,160
2002	1.1250	62,067	69,826	0.9551	7,209	6,885	1.0009	28,936	28,963	**1.0836**	97,517	105,674
United Kingdom												
1999	0.6529	1,059,532	691,812	0.5438	123,266	67,034	0.6471	238,971	154,647	**0.6427**	1,421,370	913,493
2002	0.6549	1,251,075	819,286	0.5778	141,629	81,835	0.6812	252,036	171,695	**0.6522**	1,644,859	1,072,816

In tables 5.1(a)–(c),

- the 1999 price indexes for *Domestic final demand* are spatial indexes, calculated from the component prices and volumes of all three countries in 1999 via the EKS methodology, and
- the 2002 price indexes for *Domestic final demand* are effectively temporal extrapolations of the 1999 indexes, where temporal price change is captured by a Paasche index of component prices and volumes in 1999 and 2002.

Note that in table 5.1(d) we could have opted to retain the EKS indexes for 1999, and calculated only the 2002 spatial indexes as bilateral Fisher indexes. This would be a reasonable option, as it reflects the real-time availability of the ECP data. However, we consider it more informative to keep the bilateral measurement of relative price change separate from the multilateral adjustments. Hence we conceptually introduce the transitive EKS adjustments for 1999 and 2002 simultaneously in the final stage of the reconciliation.

In Section 5.2 we look more closely at the relative importance of each stage of the reconciliation methodology, and examine a range of strategies for extracting useful insights from the process.

5.2 Analysis of results

In Section 4 we introduced the data for our case study and derived time series-based and PPP-implied measures of temporal growth in prices, volumes and expenditures. We observed the discrepancies between the time series-based and PPP-implied measures, and noted that these measures vary on the choice of reference country in our preliminary assessment of the data. In Section 5.1, we presented our interpretation of Rhoades' methodology for decomposing the discrepancy between the time series-based and PPP-implied growth measures. In this section, we return to our case study to examine the additional insights provided by the decomposition methodology. In the process we need to consider the implications for both *post hoc* and real-time analyses.

Our expectation of Rhoades' decomposition methodology is that it will provide a reliable perspective on the relative importance of component prices, component expenditures and theoretical inconsistencies in explaining the discrepancy between time series-based and PPP-implied measures of aggregate growth (i.e. at the level of *Domestic final demand*).

Having previously established that the magnitude of the discrepancy varies with the choice of reference country, we expect that the decomposition will likewise differ – perhaps significantly – according to the choice of reference country. How can we make best use of such information?

(a) Aggregate (DFD) level analysis

Table 5.2 reports the key growth measures at each step of the reconciliation process as documented in Section 5.1 and specifically in tables 5.1(a)–(d).

The first column shows the time series-based measures of temporal price change, real growth and nominal expenditure growth for *Domestic final demand* in all three countries. The final column records the corresponding PPP-implied measures of

temporal growth, using Germany as the reference country. For the reference country, we define PPP-implied measure to be equal to the time series-based measure.

Between the first and final columns, we report PPP-implied growth measures which have been extracted from tables 5.1(b)–(d), illustrating the impact of successive substitutions in explaining the discrepancy between the time series-based and PPP-implied measures. In other words, the discrepancy between the time series-based (first column) and the PPP-implied (last column) is attributed to the factors in between, namely, revisions in expenditure data (second column), price differences (third column) and index formula or theoretical (fourth column). *Therefore, our key objective in the sections to follow is to try and answer the question: what are the relative contributions to the discrepancy of the two estimates?*

The 'price discrepancies' reported at the bottom of the table are calculated as the ratio of the price measure in the same column to the PPP-implied measure in the final column, and thus monitor the progression towards total reconciliation from successive substitutions. That is, the closer they are to the value one the smaller will be the discrepancy between the time series-based and PPP-implied measures. The same method can be used to calculate 'volume discrepancies' or 'expenditure discrepancies'. This quantity is equivalent to equation (5) of the methodology section (or equations (3) and (16) in Rhoades' paper).

In table 5.2 we have intentionally ordered the sequence of substitutions so that all price and expenditure data are consistent before we attempt to assess the impact of theoretical inconsistencies. We then introduce the bilateral Fisher index, completing the information to be derived from a two-country study. The remaining discrepancy (between the final two columns) is due to the multilateral EKS adjustments. While calculation of the EKS adjustment is perhaps tractable in a three-country study, this will not be true in a wider investigation. Hence it is sensibly left to the final stage of the reconciliation process, where it can be derived as the residual.

Although we expect the order of substitutions reported in table 5.2 to provide definitive statistics on the impact of the (bilateral and multilateral) theoretical inconsistencies between the ECP and the time series statistics, we cannot be so certain about the price and expenditure effects. Thus we replicate table 5.2 by reversing the order of the substitutions, with a view to isolating the specific contributions of individual prices and expenditures. The results are shown in table 5.3.

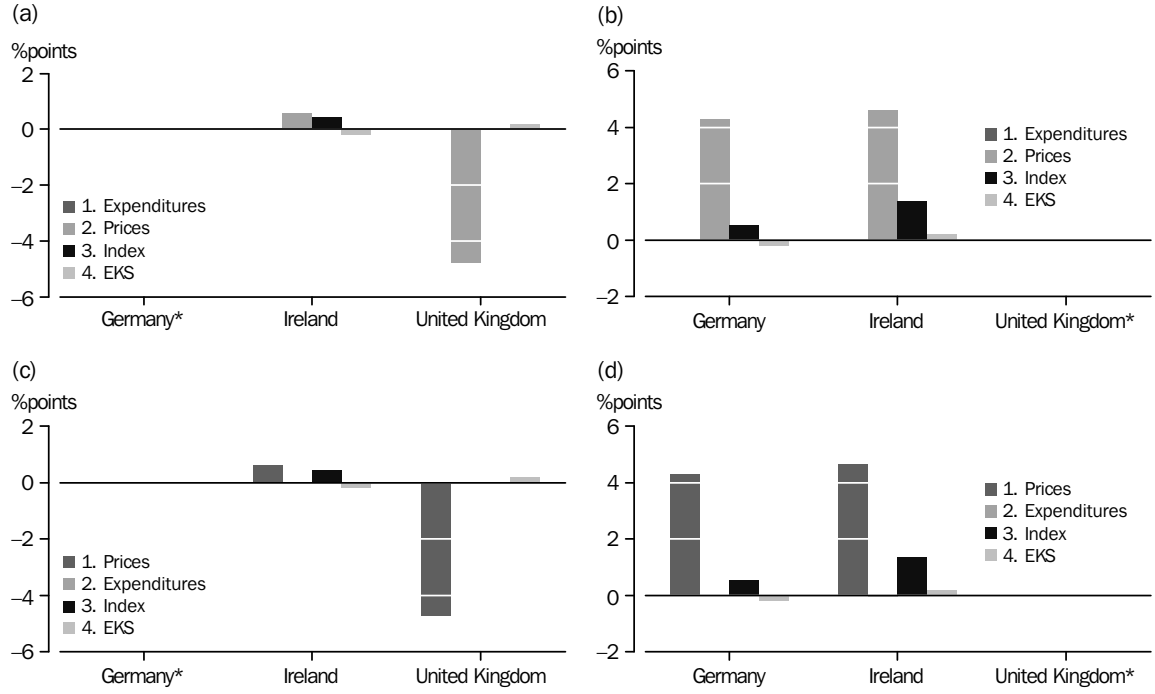
5.2 Reconciliation of 1999–2002 growth ratios for Domestic final demand

	Time series based growth	Substitution of – ECP expenditures	ECP prices	Fisher Index	PPP-implied growth
Price					
Germany*	1.0261	1.0261	1.0261	1.0261	1.0261
Ireland	1.1733	1.1736	1.1795	1.1840	1.1820
United Kingdom	1.0623	1.0625	1.0151	1.0149	1.0167
Volume					
Germany*	1.0126	1.0126	1.0126	1.0126	1.0126
Ireland	1.1802	1.1655	1.1597	1.1553	1.1573
United Kingdom	1.1066	1.1040	1.1556	1.1557	1.1538
Expenditure					
Germany*	1.0390	1.0390	1.0390	1.0390	1.0390
Ireland	1.3848	1.3679	1.3679	1.3679	1.3679
United Kingdom	1.1756	1.1730	1.1730	1.1730	1.1730
Price discrepancy					
Germany*	1.0000	1.0000	1.0000	1.0000	1.0000
Ireland	0.9927	0.9929	0.9979	1.0017	1.0000
United Kingdom	1.0449	1.0451	0.9984	0.9983	1.0000

5.3 Alternative reconciliation of 1999–2002 growth ratios for Domestic final demand

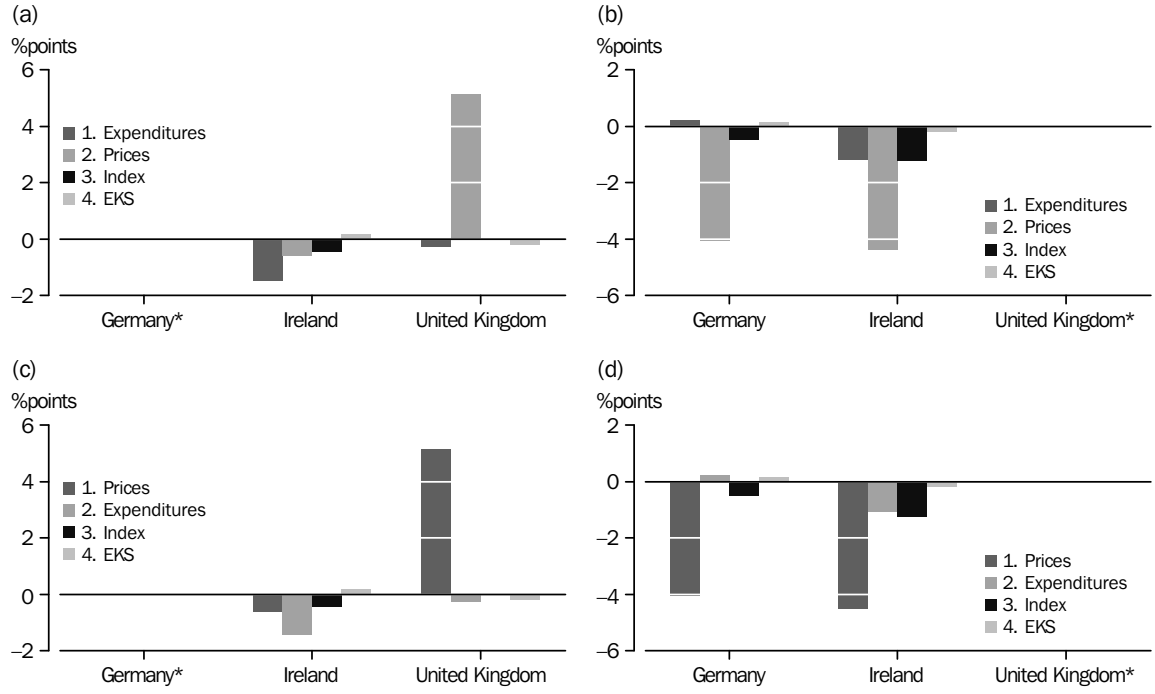
	Time series based growth	Substitution of – ECP prices	ECP expenditures	Fisher Index	PPP-implied growth
Price					
Germany*	1.0261	1.0261	1.0261	1.0261	1.0261
Ireland	1.1733	1.1795	1.1795	1.1840	1.1820
United Kingdom	1.0623	1.0150	1.0151	1.0149	1.0167
Volume					
Germany*	1.0126	1.0126	1.0126	1.0126	1.0126
Ireland	1.1802	1.1740	1.1597	1.1553	1.1573
United Kingdom	1.1066	1.1582	1.1556	1.1557	1.1538
Expenditure					
Germany*	1.0390	1.0390	1.0390	1.0390	1.0390
Ireland	1.3848	1.3848	1.3679	1.3679	1.3679
United Kingdom	1.1756	1.1756	1.1730	1.1730	1.1730
Price discrepancy					
Germany*	1.0000	1.0000	1.0000	1.0000	1.0000
Ireland	0.9927	0.9979	0.9979	1.0017	1.0000
United Kingdom	1.0449	0.9984	0.9984	0.9983	1.0000

5.4 Decomposition of the discrepancy between time series-based and PPP-implied measures of temporal price change – Domestic final demand, 1999–2002



* denotes the reference country for PPP-implied measures.

5.5 Decomposition of the discrepancy between -based and PPP-implied measures of real growth – Domestic final demand, 1999–2002



* denotes the reference country for PPP-implied measures.

Figures 5.4 and 5.5 provide an alternative visual summary of the additive (net) impact on growth rates (rather than ratios) of the four stages of reconciliation between the time series-based and PPP-implied measures of temporal price change and real growth. Note that the order of substitution is changed between parts (a) and (c), and that parts (b) and (d) report the corresponding results for the case where the United Kingdom (rather than Germany) is chosen as the reference country.

Tables D.1 and D.2 in Appendix D replicate tables 5.2 and 5.3 for the case where the United Kingdom is the reference country, and provide the source of the results presented in parts (b) and (d) of figures 5.4 and 5.5.

From figures 5.4 and 5.5, we note that the impact of expenditure substitutions is relatively minor – almost imperceptible in the case of aggregate price change. For example, the net effects of price substitution on Irish aggregate price change with the United Kingdom as a reference country is around 5 percentage points regardless of the order of substitution (see parts (b) and (d) of figures 5.4 and 5.5). On the other hand, the net effects of expenditure substitutions on aggregate price change is quite negligible, regardless of the choice of reference country or order of substitution.

Perhaps because of the insignificant role of expenditure substitutions, the impact of the component price substitutions appears virtually unaffected by the order of substitution. Of course the theoretical inconsistencies are completely unaffected by the ordering of the price and expenditure substitutions. (However, we shall observe later that the magnitude of the index substitution effect would have been quite different if we had introduced it earlier in the reconciliation sequence.) We note that the impacts on price change of the index formula and EKS adjustments are precisely balanced by the impacts on real growth.

(b) Choice of a reference country and Index effects

In Section 4, we noted that the choice of a reference country matters. That is, we have already established that PPP-implied measures referenced to Germany compare more favourably than PPP-implied measures referenced to the United Kingdom against time series-based aggregate price change and real growth in Ireland.

In tables 5.2 and 5.3 (and figures 5.4 and 5.5) we note that, perhaps due to its derivation as a multilateral correction factor, the EKS adjustment displays consistent behaviour across reference countries. For instance, when Germany is the reference country, the EKS adjustments to the growth ratios for Ireland are the reciprocals of the corresponding EKS adjustments for the United Kingdom. The same relationship holds between the EKS adjustments for Ireland and Germany when the United Kingdom is the reference country.

On the other hand, the decomposition analysis in tables 5.2 and 5.3 (and figures 5.4 and 5.5) reveal that index substitution effects vary inconsistently depending upon the reference country chosen. The most conspicuous evidence at the index substitution stage are:

- There is effectively no inconsistency between the time series-based and PPP-implied measures of temporal price change and real growth for the United Kingdom, when Germany is used as the reference country (see figures 5.4 (a) and (c) for aggregate price and figure 5.5 (a) and (c) for aggregate real growth). In the converse situation however, where the United Kingdom is used as the reference country for Germany, the substitution of the Fisher index clearly contributes to the discrepancy (see figures 5.4 (b), (d) or figures 5.5 (b),(d)).
- For subject country Ireland, we observe that the index substitution effect is considerably larger when the United Kingdom is the reference country, being of comparable magnitude to and amplifying the price substitution effect – rather than partially offsetting the price effect as in the case where Germany is the reference country.

To summarise our points, we have found a relationship that links the net discrepancies based on different reference countries, and we can understand the behaviour of the EKS adjustments, but we cannot identify any consistency in the relative contributions of the price, expenditure and index formula effects. We also note that, for any given choice of reference country, the relative contributions of the price, expenditure and index formula effects vary according to the order in which they are implemented.

As there exists no unique decomposition of the price, expenditure and index formula effects, we suggest it may be more practical to consider each effect separately. Specifically, we suggest quantifying each effect conditional upon all other substitutions having been made. We then propose to examine the 'conditional' effects based upon different reference countries to discover whether there are any similarities or patterns.

(c) Aggregation methodology in real-time analysis

We have noted already that the order of substitutions documented in tables 5.2 and 5.3 was designed to facilitate analysis of the index substitution effect, conditional upon all other substitutions having been made. We have also noted that the impact of the index substitution on the measurement of aggregate price change is the reciprocal of the impact on real growth, and that the index substitution effect under one choice of reference country appears unrelated to the effect observed when an alternative reference country is employed. The presence of sizable index effects in the 1999–2002 analysis indicates at least the potential for future errors.

Our case study suggests that for future analysis of Irish PPPs, we would be unwise to use deflators at a high aggregate level such as *Domestic final demand* deflators from the national accounts if the alternative of using disaggregated data were available.

Thus, in a real-time analysis, it would be imperative for practitioners to employ the Fisher index formulae in their calculations from the outset, seeking to eliminate any prospect of theoretical inconsistency. It is likely, however, that data availability will determine the level of disaggregation achievable.

In general, it would be unwise to project PPPs with an aggregate Paasche price index where we suspect that component price movements are likely to be heterogeneous. The risk from heterogeneous price movements might perhaps be reduced by selecting a reference country which is experiencing similar component price movements, in the expectation that the resulting spatial price relatives may be more homogeneous. The scope for inconsistencies due to choice of index formula would thus be reduced.

In Appendix E, we explore the index substitution effect in greater mathematical detail, examining the role of expenditure weights and spatial price relatives at the sub-aggregate level. We show that 'theoretical' inconsistencies can only occur when there is significant heterogeneity in (bilateral) spatial price relatives at the sub-aggregate level, although it is temporal changes in the expenditure weights for the two countries that determine the magnitude of the effect.

(d) Sub-aggregate level price substitution effects

The sequence of substitutions for determining the price substitution effects at the sub-aggregate level is set out in table 5.6. This sequence is perhaps quite plausible for a real-time investigation. An analyst may choose to replicate the PPP index methodology from the outset, and may have real-time knowledge of necessary corrections to the expenditure estimates. The national accounts, however, provide the only source of relevant price data.

A more specific description of our hypothetical scenario is the following:

We have ECP-based data on all component expenditures for the subject and reference country, and know the 1999 benchmark PPPs for all sub-aggregates. We estimate PPPs for 2002 by extrapolating the 1999 benchmarks by time series-based implicit price deflators (Paasche Index). Spatial price indexes (PPPs) for *Domestic final demand* in 1999 and 2002 are computed using the Fisher index formula. We then derive a measure of PPP-implied temporal price change for *Domestic final demand*. Finally, we compare this measure with the result obtained by using *actual* 2002 benchmark PPPs rather than the extrapolated estimates.

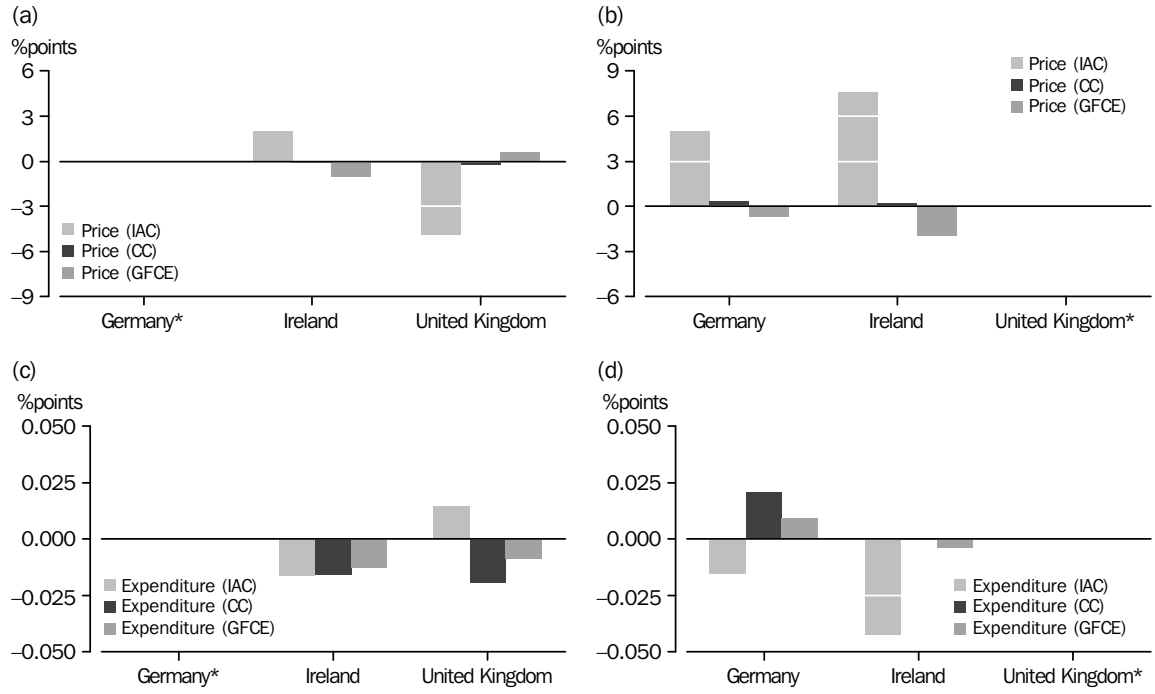
5.6 Reconciliation of 1999–2002 growth ratios for Domestic final demand – Price effects

	Substitution of –						
			ECP prices				
	Time series based growth	Fisher Index	ECP expenditures	IAC	CC	GFCE	PPP-implied growth
Price							
Germany*	1.0261	1.0261	1.0261	1.0261	1.0261	1.0261	1.0261
Ireland	1.1733	1.1760	1.1757	1.1955	1.1941	1.1840	1.1820
United Kingdom	1.0623	1.0607	1.0605	1.0115	1.0086	1.0149	1.0167
Volume							
Germany*	1.0126	1.0126	1.0126	1.0126	1.0126	1.0126	1.0126
Ireland	1.1802	1.1776	1.1634	1.1441	1.1455	1.1553	1.1573
United Kingdom	1.1066	1.1083	1.1061	1.1596	1.1630	1.1557	1.1538
Expenditure							
Germany*	1.0390	1.0390	1.0390	1.0390	1.0390	1.0390	1.0390
Ireland	1.3848	1.3848	1.3679	1.3679	1.3679	1.3679	1.3679
United Kingdom	1.1756	1.1756	1.1730	1.1730	1.1730	1.1730	1.1730
Price discrepancy							
Germany*	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Ireland	0.9927	0.9949	0.9947	1.0115	1.0103	1.0017	1.0000
United Kingdom	1.0449	1.0433	1.0431	0.9950	0.9921	0.9983	1.0000

5.7 Reconciliation of 1999–2002 growth ratios for Domestic final demand – Expenditure effects

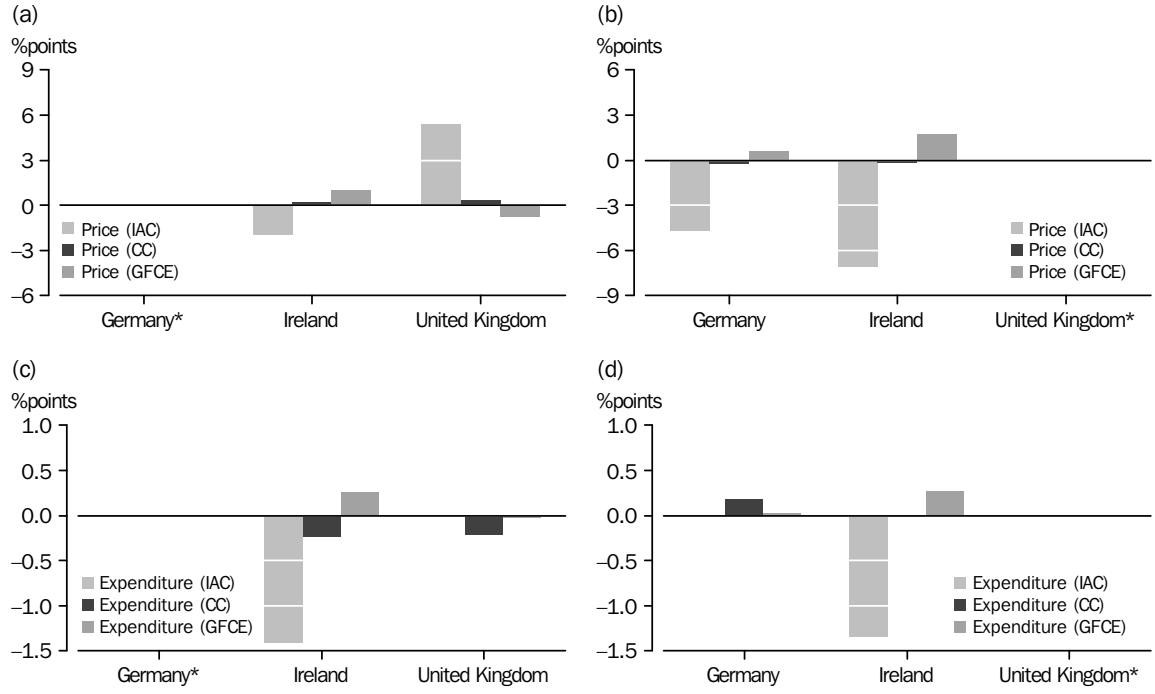
	Substitution of –						
			ECP expenditures				
	Time series based growth	Fisher Index	ECP prices	IAC	CC	GFCE	PPP-implied growth
Price							
Germany*	1.0261	1.0261	1.0261	1.0261	1.0261	1.0261	1.0261
Ireland	1.1733	1.1760	1.1844	1.1843	1.1841	1.1840	1.1820
United Kingdom	1.0623	1.0607	1.0151	1.0152	1.0150	1.0149	1.0167
Volume							
Germany*	1.0126	1.0126	1.0126	1.0126	1.0126	1.0126	1.0126
Ireland	1.1802	1.1776	1.1692	1.1551	1.1528	1.1553	1.1573
United Kingdom	1.1066	1.1083	1.1581	1.1581	1.1560	1.1557	1.1538
Expenditure							
Germany*	1.0390	1.0390	1.0390	1.0390	1.0390	1.0390	1.0390
Ireland	1.3848	1.3848	1.3848	1.3679	1.3650	1.3679	1.3679
United Kingdom	1.1756	1.1756	1.1756	1.1757	1.1733	1.1730	1.1730
Price discrepancy							
Germany*	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Ireland	0.9927	0.9949	1.0021	1.0019	1.0018	1.0017	1.0000
United Kingdom	1.0449	1.0433	0.9984	0.9986	0.9984	0.9983	1.0000

5.8 Contribution of price and expenditure effects to the discrepancy between time series-based and PPP-implied measures of temporal price change – Domestic final demand, 1999–2002



* denotes the reference country for PPP-implied measures.

5.9 Contribution of price and expenditure effects to the discrepancy between time series-based and PPP-implied measures of real growth – Domestic final demand, 1999–2002



* denotes the reference country for PPP-implied measures.

Whether the price effect is small or large, there are likely to be significant insights to be gained by examining the contributions from the sub-aggregate price effects. For example, it will be useful to know whether a large price effect can be traced to a single source, or whether there is some uniform level of bias present in the time series-based PPP extrapolations. Similarly, it is valuable to know whether a small net price effect is the result of a fortuitous balancing of larger component effects.

Table 5.6 illustrates the calculation of the sub-aggregate price effects, and parts (a) and (b) of figures 5.8 and 5.9 illustrate the results. (See also table D.3 in Appendix D.)

The first observation we should make regarding sub-aggregate price effects is that the impact of substituting the ECP price for *Individual actual consumption* will differ depending upon whether or not the ECP prices for *Collective consumption* and *Gross fixed capital expenditure* have already been substituted. That is, the effects are order-dependent, and strictly we should measure each effect conditional upon all other component prices having been substituted. In practice, the distortion in our current presentation is very small.

In Section 4 we noted that component price substitutions would have reciprocal effects on component volume measures (as expenditures are fixed). These reciprocal effects on aggregate price change and real growth are clearly apparent when we compare parts of (a) and (b) of figures 5.8 and 5.9, respectively.

In our specific case study, the price substitution effects for individual consumption differ both in magnitude and direction as compared to the effects of substituting the price of capital (see figures 5.8 and 5.9 parts (a) and (b)). Both effects are also sufficiently large to suggest some incompatibility between the time series and ECP collections. The price of collective consumption does not play a significant role.

By contrasting the results in parts (a) and (b) of figures 5.8 and 5.9, we can see the effects of the choice of the reference country. For Ireland, we observe that not only are the relative contributions of the three components inconsistent in magnitude and sometimes direction (in the case of *Collective consumption*) for a given reference country but also the direction of the overall price substitution effect differs with the choice of reference country.

In comparing (a) and (b) of figures 5.8 and 5.9, we observe that the price substitution effects for Germany with the United Kingdom as reference country are the inverses of the effects for the United Kingdom with Germany as reference country.

Taking the individual sub-aggregates separately, we find that there is a consistent relationship between the price substitution effects (expressed as growth ratios) in the three countries. For example, the impact of substituting the ECP price for *Individual actual consumption* is greater in Ireland than in Germany, and greater in Germany than in the United Kingdom – regardless of the reference country employed.

(e) Sub-aggregate expenditure substitution effects

In real time, it seems improbable that ECP price data would be available prior to ECP expenditure data. Hence we view our methodology as primarily a *post hoc* device for obtaining a relative measure of the significance of the discrepancy between time series and ECP expenditure estimates.

The sequence of substitutions for determining the expenditure substitution effects at the sub-aggregate level is set out in table 5.7. Parts (c) and (d) of figures 5.8 and 5.9 illustrate these results. Again, from tables 5.6 and 5.7 (and figures 5.8 and 5.9) there is very little evidence to suggest that either the price substitution effect or the expenditure substitution effect is altered appreciably by changing the order of implementation. The overall impact of substituting ECP expenditures remains very small, whether we consider temporal price movements or real growth. Note also that we have employed different vertical scales for the expenditure effects in figures 5.8 and 5.9.

In general, as changes to expenditure have a direct proportional impact on real growth, there is always the prospect that a large time series–ECP discrepancy may result in a large revision to PPP-implied real growth. As changes to expenditure can only impact on aggregate prices indirectly via changes to weighting patterns, we expect such impacts to be minor. Certainly the symmetry that we observed for price substitution effects is not likely to occur for expenditure substitution effects. Indeed we observe that the effects on temporal price change and real growth are not even offsetting in all cases.

As for the price substitutions, the expenditure substitution effects for Germany with the United Kingdom as reference country are the inverses of the effects for the United Kingdom with Germany as reference country. However, this appears to be the only consistent pattern to be found in figures 5.8 and 5.9.

6. CONCLUSIONS

Given the unavoidable costs associated with conducting extensive international comparisons of prices, continuing benefits may be expected from any methodology capable of producing timely and accurate projections of Purchasing Power Parities. Rhoades' concept of PPP-implied growth appears to offer a useful perspective for analysing variants of the projection methodology – the use of time series-based price movements to extrapolate benchmark PPPs.

PPP-implied measures of price change and real growth essentially provide bilateral comparisons of the coherence between the time series-sourced and PPP-sourced statistics of a subject country and a reference country. For a given subject country, the PPP-implied growth measures may vary enormously as the reference country changes, which implies that the PPP-implied measures are not reference country invariant.

Where the PPP-implied measures differ greatly from their corresponding time series-based measures, we may deduce that the time series-based projection methodology is probably unsatisfactory. However, it should not automatically be assumed that close proximity of the PPP-implied and time series-based measures validates the methodology.

Rhoades' decomposition methodology, which we have sought to explain and understand in Sections 2, 4 and 5, provides a means for more thorough validation. In our case study for example, we observed good coherence between the time series-based estimate of aggregate price change in Ireland and the PPP-implied price change measure referenced to Germany. However more detailed analysis showed substantial, but offsetting, discrepancies associated with prices at the sub-aggregate level. Moreover, the net price inconsistency was partially masked by index formula effects.

Although Rhoades' decomposition of the discrepancy into price, expenditure and index formula substitution effects is not orthogonal or unique, our 'conditional' effects appear to have good practical application for *post hoc* analysis.

Price substitution effects, which we expect will generally be the dominant effects, will be useful for identifying countries which have coherent PPPs and time series of prices. Although the choice of reference countries may be largely predetermined (e.g. to major trading partners) there may be advantages to analysing clusters of countries with similar characteristics. Also, unanticipated dissimilarities may highlight weaknesses in the statistical collection process.

We also note that the significance of the choice of reference country in our analysis indicates that further work is needed in relation to the underlying assumptions that may be taken into consideration in choosing a reference country (such as similarities

in countries' economic structures, price movements and the evolution of economic growth paths) in order to provide a framework upon which a suitable reference country (countries) can be considered in the analysis.

The discovery of large index formula substitution effects provides a clear indication that the disaggregated analysis was warranted. In a more comprehensive analysis, this criterion may be useful in establishing 'practical' levels of disaggregation for future projections.

In particular, we identified significant discrepancies in GFCE possibly due to significant differences in the statistical methods employed to value capital goods in the three countries. For example, Germany and the United Kingdom use hedonics to produce the deflators for computers in their investment series but not in Ireland. An important implication of this result is that the conventional view that PPP data is always inferior to the time series collections is incorrect.

We have suggested a couple of ways in which Rhoades' methodology may be used in *post hoc* analysis to inform the calculation of future PPP projections (assuming the findings remain robust over time). As most of the effects analysed in this methodology are in fact unobservable in real time, we must conclude that past analysis provides the best guide for real-time analysis.

In relation to aggregation methodology, our empirical analysis suggests that we will be better off using the spatial Fisher indexes for all aggregation tasks, and therefore minimising the potential for theoretical inconsistencies to affect the outcome. The level of disaggregation should also be designed to avoid the theoretical inconsistencies observed in past analyses. Comparing relative price movements in countries previously identified as similar may also be informative.

None of these recommendations are particularly radical or innovative, but perhaps the adoption of the PPP-implied growth paradigm may serve to link practical diagnostics with future modifications to PPP projections.

7. NOTES FROM THE METHODOLOGY ADVISORY COMMITTEE

This paper was presented and discussed at the ABS Methodology Advisory Committee (MAC) meeting on 17 November 2006. The following are a summary of the main comments and suggestions made on this research paper:

- The discussions and conclusions in the paper are relative comparisons and they are not reference country invariant. It is important that this fact be clearly stated in the conclusion.
- The number of issues dealt with in this research paper are numerous. To support the discussion on these complex and interrelated issues, the paper presents numerous tables and graphs. The mere size of these tables and figures makes the paper not so easy to follow the arguments and discussions on each issue. The paper could have benefited by focussing on one or two issues only.
- Overall the paper makes important contributions in its own right in identifying the sources of inconsistency between spatial and temporal comparisons. The conceptual framework used in this analysis has been employed correctly and provides useful empirical insights. Of course, this would have been even more interesting if there had been some kind of economic theory to guide the analysis; a question which does not seem to have an easy answer in sight at least at the moment.
- The relative contributions of expenditure, prices and index formula are order-dependent. One problem with this is that there are a number of different ways we can order such effects. Nevertheless, the paper also identifies that the relative contributions of prices remained as a dominant factor no matter what order we use in the analysis. This is an important finding and it would be interesting to undertake follow up research using more disaggregated data sets and see the prices of which items are relatively more important than others.
- The paper suggests the use of the *Fisher index* for all aggregation purposes in order to minimise the potential for theoretical inconsistencies. This is an important finding and is worth highlighting as one of the paper's main findings.
- The issue of *hedonics* was discussed as an example of potential sources of discrepancy in GFCE in Ireland as compared to Germany and the United Kingdom. If that is the case this could have important implications for international comparisons and needs to be highlighted as such.

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APPENDIXES

A. RHOADES' ANALYTICAL FRAMEWORK

Rhoades (2003) proposed a spatial–temporal framework which can be used to decompose the relative contribution of the factors such as price changes, changes in weights or EKS adjustment, or a combination of any of these to the inconsistency of PPP-based and time series-based estimates of real growth. The framework is based on the following 'intuitive' definition.

> "If country *S* is 10% that of country *R* in 1996 and 12% in 2000, then this implies that *S* grew by 20 percent over the period. PPP implied real growth is then given by adding *R*'s real growth to the 20%."

Rhoades' framework is summarized below:

Let the symbols *S* and *R* represent the subject and reference country, respectively.

Define

$$GDP_t^S = \sum_i p_{it}^S q_{it}^S \quad \text{and} \quad GDP_t^R = \sum_i p_{it}^R q_{it}^R$$

as the national accounts-based estimates for country *S* and *R* , respectively, at time *t*, where *i* stands for the i^{th} expenditure category.

Similarly, let the price ratios

$$\frac{p_{it}^S}{p_{it}^R} \quad \text{and} \quad \frac{p_{it+k}^S}{p_{it+k}^R}$$

represent the binary Fisher estimates of PPP for the i^{th} expenditure category at time *t* and $t+k$. These PPP can be calculated based on two sources; directly from the ECP program (ECP-based) or indirectly from the expenditure data of the time series-based System of National Accounts (SNA). For convenience, let the respective ECP-based estimates be denoted by

$$\frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \quad \text{and} \quad \frac{\tilde{p}_{it+k}^S}{\tilde{p}_{it+k}^R}$$

Note that the symbol '∼' will be used through out our discussion to distinguish the ECP-based estimates from the other.

At the GDP level the EKS procedure is applied to achieve transitivity across countries. That is, the PPP for country *S* relative to *R* is defined as:

$$PPP_t^{S(R)} = \left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} \left(\sum_i w_{it}^R \frac{\tilde{p}_{it}^S}{\tilde{p}_{it}^R} \right)^{1/2} + \varepsilon_t^{S(R)} \qquad (1)$$

where $w_{it}^{S(R)}$ and w_{it}^{R} represent value shares from the national accounts estimates for the i^{th} expenditure category for countries S and R, respectively, and are defined as

$$w_{it}^{R} = \frac{p_{it}^{R} q_{it}^{R}}{\sum_i p_{it}^{R} q_{it}^{R}}$$

and

$$w_{it}^{S(R)} = \frac{p_{it}^{R} q_{it}^{S}}{\sum_i p_{it}^{R} q_{it}^{S}}$$

$\varepsilon_t^{S(R)}$ represents the combined adjustment introduced by the EKS and linking at the GDP level.

Thus GDP for S in R's currency at time t is given by:

$$GDP_t^{S(R)} = \sum_i p_{it}^{S} q_{it}^{S} \left[\left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} \left(\sum_i w_{it}^{R} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} + \varepsilon_t^{S(R)} \right]^{-1} \qquad (2)$$

Once GDP for S is converted to R's currency, the real growth in S, as implied by PPP, can be expressed as the real growth in the reference country R multiplied by the growth in GDP proportion of S relative to R. That is,

$$INT_{t+k} = \frac{\sum_i p_{it}^{R} q_{it+k}^{R}}{\sum_i p_{it}^{R} q_{it}^{R}} \frac{\dfrac{\sum_i p_{it+k}^{S} q_{it+k}^{S}}{\sum_i p_{it+k}^{R} q_{it+k}^{R}}}{\dfrac{\sum_i p_{it}^{S} q_{it}^{S}}{\sum_i p_{it}^{R} q_{it}^{R}}} \frac{\left[\left(\sum_i w_{it+k}^{S(R)} \frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}} \right)^{1/2} \left(\sum_i w_{it+k}^{R} \frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}} \right)^{1/2} + \varepsilon_{t+k}^{S(R)} \right]^{-1}}{\left[\left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} \left(\sum_i w_{it}^{R} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} + \varepsilon_t^{S(R)} \right]^{-1}} \qquad (3)$$

Multiplying and dividing equation (3) by the real growth in S between periods t and $t+k$ and rearranging terms gives:

$$INT_{t+k} = \frac{\sum_i p_{it}^{S} q_{it+k}^{S}}{\sum_i p_{it}^{S} q_{it}^{S}} \frac{\dfrac{\sum_i p_{it+k}^{S} q_{it+k}^{S}}{\sum_i p_{it}^{S} q_{it+k}^{S}}}{\dfrac{\sum_i p_{it+k}^{R} q_{it+k}^{R}}{\sum_i p_{it}^{R} q_{it+k}^{R}}} \frac{\left[\left(\sum_i w_{it+k}^{S(R)} \frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}} \right)^{1/2} \left(\sum_i w_{it+k}^{R} \frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}} \right)^{1/2} + \varepsilon_{t+k}^{S(R)} \right]^{-1}}{\left[\left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} \left(\sum_i w_{it}^{R} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}} \right)^{1/2} + \varepsilon_t^{S(R)} \right]^{-1}} \qquad (4)$$

where

$$w_{it+k}^{R} = \frac{p_{it+k}^{R} q_{it+k}^{R}}{\sum_i p_{it+k}^{R} q_{it+k}^{R}}$$

and

$$w_{it+k}^{S(R)} = \frac{p_{it+k}^{R} q_{it+k}^{S}}{\sum_i p_{it+k}^{R} q_{it+k}^{S}}$$

Therefore the intuitive growth rate equals the real growth rate in *S*, times the ratio of the implicit deflator in *S* relative to *R*, divided by the ratio of PPPs at time $t+k$ relative to time t.

> "Note that if the PPP is estimated by the ratio of implicit deflators, as is often done for the purpose of projecting PPPs between benchmarks, then INT_{t+k} will exactly equal real growth in *S*. This gives us some confidence that INT_{t+k} is a sensible concept despite the problems associated with its interpretation when actual benchmark PPPs are used in its calculation instead of deflator-based projections." (Rhoades, 2003, p. 7).

The difference between intuitive ECP-implied growth INTt+k and the time series-based growth $\frac{\sum_i p_{it}^{S} q_{it+k}^{S}}{\sum_i p_{it}^{S} q_{it}^{S}}$ is embedded in the ratio of implicit deflators divided by the ratio of PPPs.

Let's rewrite equation 4 as:

$$INT_{t+k} = \frac{\sum_i p_{it}^{S} q_{it+k}^{S}}{\sum_i p_{it}^{S} q_{it}^{S}} \times \Delta_{t+k}$$

where

$$\Delta_{t+k} = \frac{\dfrac{\sum_i p_{it+k}^{S} q_{it+k}^{S}}{\sum_i p_{it}^{S} q_{it+k}^{S}}}{\dfrac{\sum_i p_{it+k}^{R} q_{it+k}^{R}}{\sum_i p_{it}^{R} q_{it+k}^{R}}} \frac{\left[\left(\sum_i w_{it+k}^{S(R)} \frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}}\right)^{1/2}\left(\sum_i w_{it+k}^{R} \frac{\tilde{p}_{it+k}^{S}}{\tilde{p}_{it+k}^{R}}\right)^{1/2} + \varepsilon_{t+k}^{S(R)}\right]^{-1}}{\left[\left(\sum_i w_{it}^{S(R)} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}}\right)^{1/2}\left(\sum_i w_{it}^{R} \frac{\tilde{p}_{it}^{S}}{\tilde{p}_{it}^{R}}\right)^{1/2} + \varepsilon_{t}^{S(R)}\right]^{-1}} \qquad (5)$$

As stated earlier the difference between the ECP-based (PPP-implied) and time series-implied growth is determined by Δ_{t+k}. Given this expression, no difference between the ECP-based and time series-based growth rates is implied by a value of Δ_{t+k} which is equal to one. On the other hand a value greater than one indicates the former is greater than the later while less than one indicates the vice versa. That is, the farther it is from unity the larger the discrepancy between the spatial PPP-based estimates and the time series-based real growth rates.

In order to assess the contributions of the various components into the discrepancy of the estimates Rhoades shows that the above expression can be decomposed into a detailed set of contributors as follows (See Rhoades (2003) for further details and related assumptions).

A.1 Impact of differences in basic price data

The impact of basic input data can be investigated by rewriting equation (5) as:

$$\Delta_{t+k} = \frac{\sum_i W^R_{it+k} \frac{p^S_{it+k}}{p^R_{it+k}}}{\sum_i W^{R(t)}_{it+k} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}} \frac{\left[\left(\sum_i \tilde{w}^{S(R)}_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}} \right)^{1/2} \left(\sum_i \tilde{w}^R_{it+k} \frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}} \right)^{1/2} + \varepsilon^{S(R)}_{t+k} \right]^{-1}}{\left[\left(\sum_i \tilde{w}^{S(R)}_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}} \right)^{1/2} \left(\sum_i \tilde{w}^R_{it} \frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}} \right)^{1/2} + \varepsilon^{S(R)}_{t} \right]^{-1}} \qquad (6)$$

where the various components are defined as follows:

$$W^R_{it+k} = w^R_{it+k} \left(\frac{\tilde{p}^R_{it}}{\tilde{p}^S_{it}} \right) \frac{p^S_{it} q^S_{it+k}}{p^R_{it} q^R_{it+k}}$$

$$W^{R(t)}_{it+k} = w^{R(t)}_{it+k} \left(\frac{\tilde{p}^R_{it}}{\tilde{p}^S_{it}} \right) \frac{p^S_{it} q^S_{it+k}}{p^R_{it} q^R_{it+k}}$$

and

$$w^{R(t)}_{it+k} = \frac{p^R_{it}\, q^R_{it+k}}{\sum_i p^R_{it}\, q^R_{it+k}}$$

Note that the symbol tilde (~) is used to indicate estimates based on national accounts at the time where the PPPs were compiled. Those without are national accounts estimates after subsequent revisions have been made.

Thus, depending on the estimates, the values can be calculated either from the national accounts at the time where the PPPs were calculated or from a subsequently revised national accounts.

Impacts of changes in price data are assessed by substituting time series-based prices in the left hand side of the numerator in the equation by ECP-based prices.

A.2 Impact of different treatment of weights

The impact of changes in weights can be assessed by expressing the above equation as:

$$\Delta_{t+k} = \frac{\left(\sum_i W^R_{it+k}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i W^R_{it+k}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left[\left(\sum_i \tilde{w}^{S(R)}_{it+k}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i \tilde{w}^{R}_{it+k}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2} + \varepsilon^{S(R)}_{t+k}\right]^{-1}}{\left(\sum_i W^{R(t)}_{it+k}\frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i W^{R(t)}_{it+k}\frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left[\left(\sum_i \tilde{w}^{S(R)}_{it}\frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}\left(\sum_i \tilde{w}^{R}_{it}\frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2} + \varepsilon^{S(R)}_{t}\right]^{-1}} \quad (7)$$

Again one can substitutive sequentially ECP-based weights for time series-based weights into this expression to assess the impacts on changes in index formulas.

A.3 Impact of EKS adjustment

After making the weight substitutions equation 7 becomes:

$$\Delta_{t+k} = \frac{1+\varepsilon^{S(R)}_{t}\left(\sum_i \tilde{w}^{S(R)}_{it+k}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i \tilde{w}^{R}_{it+k}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}}{1+\varepsilon^{S(R)}_{t+k}\left(\sum_i \tilde{w}^{S(R)}_{it}\frac{\tilde{p}^S_{it+k}}{\tilde{p}^R_{it+k}}\right)^{1/2}\left(\sum_i \tilde{w}^{R}_{it}\frac{\tilde{p}^S_{it}}{\tilde{p}^R_{it}}\right)^{1/2}} \quad (8)$$

Thus if the EKS adjustments are zero equation 8 will be equal to one (see Rhoades (2003) for further details and discussions.

B. NOMINAL EXPENDITURES IN NATIONAL CURRENCIES FOR THE YEARS 1999 AND 2002

B.1 Nominal expenditure in national currencies for the years 1999 and 2002

	1999				*2002*			
Country	*IAC*	*CC*	*GFCE*	*DFD*	*IAC*	*CC*	*GFCE*	*DFD*
Austria	135,628	15,410	45,535	196,573	149,209	15,383	48,322	212,913
Belgium	158,285	18,423	49,308	226,016	178,124	21,446	51,558	251,128
Germany	1,375,800	158,340	426,390	1,960,530	1,479,550	168,100	391,760	2,039,410
Denmark	109,269	13,334	32,115	154,718	120,766	13,863	37,805	172,434
Spain	391,988	41,584	136,337	569,909	476,225	52,835	175,356	704,416
Finland	76,668	9,612	23,492	109,772	90,761	10,681	26,572	128,014
France	931,573	126,474	260,407	1,318,454	1,054,258	142,400	301,091	1,497,749
Greece	86,297	10,623	25,529	122,449	103,618	13,325	33,779	150,722
Ireland	50,949	4,802	21,409	77,160	69,826	6,885	28,963	105,674
Iceland	5,899	594	1,781	8,275	6,418	742	1,703	8,863
Italy	788,560	78,926	210,623	1,078,108	906,024	90,287	249,263	1,245,574
Luxembourg	9,771	1,288	4,458	15,517	12,020	1,659	4,994	18,673
Netherlands	232,508	40,611	84,186	357,305	280,115	50,640	92,155	422,910
Norway	89,943	12,098	32,709	134,750	116,971	18,446	35,869	171,286
Portugal	80,137	8,511	29,463	118,111	95,556	10,704	31,886	138,146
Sweden	160,267	19,706	40,684	220,657	174,889	21,853	42,722	239,464
Switzerland	166,870	12,023	55,390	234,282	195,265	16,579	59,984	271,828
United Kingdom	691,812	67,034	154,647	913,493	819,286	81,835	171,695	1,072,816

Source: Eurostat

C. SOME USEFUL REFERENCE COUNTRY INVARIANCE PROPERTIES

As noted in Section 4, a closer examination of the results in table 4.1 reveal a consistency between estimates of PPP-implied price change in the three countries which is not distorted by the choice of reference country. To illustrate it empirically:

Converting the percentage growth rates in table 4.1 to *growth ratios*:

$$\textit{growth ratio} \;=\; 1+\frac{\%\textit{growth rate}}{100}$$

we observe a useful relationship between growth ratios which is invariant to the choice of reference country:

$$1.0261:1.1820:1.0167 \;=\; 1.0185:1.1733:1.0092 \;=\; 1.0722:1.2351:1.0623$$

$$=\; 0.8681:1.0000:0.8601$$

That is, the relative growth ratio pertaining to aggregate price change for Germany is 86.81% of the growth ratio for Ireland, and similarly the growth ratio for the United Kingdom is 86.01% of the growth ratio for Ireland – regardless of the reference country chosen.

If the relative growth ratios are invariant to the choice of reference country, but the actual growth ratios differ, it is logical to assume that either the United Kingdom or Germany will be 'closer' than the other to our chosen subject country of Ireland.

In fact, from the columns of table 4.1, we can observe that

$$1.0261:1.0185:1.0722 \;=\; 1.1820:1.1733:1.2351 \;=\; 1.0167:1.0092:1.0623$$

$$=\; 1.0074:1.0000:1.0526$$

That is, regardless of the subject country chosen, growth ratio estimates of PPP-implied price change based upon Germany as the reference country will be 0.74% higher than estimates based upon Ireland, while growth ratio estimates referenced to the United Kingdom will be 5.26% higher.

Hence Germany represents the better choice of reference country for Ireland based upon this *post hoc* analysis of the data. This fact could, of course, have been deduced trivially from a comparison of figures 4.2(a) and 4.2(b). The invariance results provided above, however, suggest the possibility that a superior result might be feasible – perhaps as a consequence of weighting together reference countries.

If, hypothetically, PPP-implied price change in all countries was proportionally a little higher than in figure 4.2(a) and a little lower than in figure 4.2(b), then

- PPP-implied price change in Ireland would equal the time series-based measure, and

- The bilateral comparison of price change measures in Germany and the United Kingdom would also be improved.

The invariance properties described for PPP-implied aggregate price change also apply to PPP-implied real growth:

- The growth ratio pertaining to real growth in Germany is 87.49% of the growth ratio for Ireland, and the growth ratio for the United Kingdom is 99.70% of the growth ratio for Ireland – regardless of the reference country chosen; and
- Growth ratio estimates of PPP-implied real growth based upon Germany as the reference country will be 1.94% higher than estimates based upon Ireland, and growth ratio estimates referenced to the United Kingdom will be 5.95% lower.

Note from the latter result that Germany is again the better choice of reference country for Ireland – see also figures 4.2(c) and 4.2(d).

The invariance properties we noted for *Domestic final demand*, in our discussion of table 4.1, apply also to the three sub-aggregates. For example, the relationship between growth ratios pertaining to PPP-implied price change in *Gross fixed capital expenditure* for the three countries in our case study is 0.8463 (Germany) : 1.0000 (Ireland) : 0.9054 (United Kingdom) – regardless of the choice of reference country (refer table C.1). Similarly, PPP-implied price measures referenced to Germany will be marginally closer to Irish measures than PPP-implied measures referenced to the United Kingdom, in accordance with the fixed ratio

0.9642 (Germany) : 1.0000 (Ireland) : 0.9308 (United Kingdom)

C.1 Measures of temporal price change – Gross fixed capital expenditure, 1999–2002

	Germany	*Ireland*	*United Kingdom*
Time series-based	–1.61%	20.58%	1.62%
PPP-implied (Reference=Germany)	–1.61% *	16.27%	5.27%
PPP-implied (Reference=Ireland)	2.04%	20.58% *	9.17%
PPP-implied (Reference=United Kingdom)	–5.01%	12.24%	1.62% *

Unfortunately, there is no invariance property linking the discrepancies observed for the three sub-aggregates. Hence time series-based and PPP-implied measures may be similar for one component but not another, and Germany may be the best choice of reference country for one component and the United Kingdom may be best for another. We suggest only that if Germany is unambiguously the best choice of reference country for *Domestic final demand*, then it is likely to also be the best choice for most of the sub-aggregates (as indeed is the case here).

Given the high rates of both price inflation and real growth reported by Ireland between 1999 and 2002, the discrepancies we observe in figure 4.3 between the time series-based and PPP-implied measures for IAC and CC (based upon Germany) are perhaps not unduly worrisome. However, the time series-based and PPP-implied measures for GFCE are totally incompatible (as is also evident from table C.1).

On the basis of time series data, the price of capital goods in Ireland appears to have risen much faster than the price of consumption goods, and consequently real growth in GFCE lags considerably behind real growth in consumption. By contrast, the time-series data for Germany and the United Kingdom show growth in their GFCE price deflators to be much lower than growth in their corresponding consumption deflators. In fact the German GFCE deflator suggests that the price of capital fell between 1999 and 2002. The PPP-implied measures for Ireland (based upon *both* reference countries) also support this converse scenario, reporting very low growth in the price of capital.

We suggest that these conflicting scenarios may be attributed to

- significant differences between the mix of capital goods purchased in Ireland and the capital goods purchased in Germany and the United Kingdom, and/or
- significant differences in the statistical methods employed to value capital goods in the three countries.

In subsequent sections, we shall demonstrate how the observed inconsistencies between the PPP and time-series data on component prices and expenditures influence the discrepancy between PPP-implied and time series-based measures of aggregate price change and real growth.

D. DECOMPOSITION RESULTS

D.1 Reconciliation of 1999–2002 growth ratios for Domestic final demand

		Substitution of –			
	Time series based growth	*ECP expenditures*	*ECP prices*	*Fisher Index*	*PPP-implied growth*
Price					
Germany	1.0261	1.0259	1.0686	1.0740	1.0722
Ireland	1.1733	1.1734	1.2194	1.2330	1.2351
United Kingdom*	1.0623	1.0623	1.0623	1.0623	1.0623
Volume					
Germany	1.0126	1.0150	0.9745	0.9695	0.9712
Ireland	1.1802	1.1683	1.1243	1.1119	1.1100
United Kingdom*	1.1066	1.1066	1.1066	1.1066	1.1066
Expenditure					
Germany	1.0390	1.0413	1.0413	1.0413	1.0413
Ireland	1.3848	1.3709	1.3709	1.3709	1.3709
United Kingdom*	1.1756	1.1756	1.1756	1.1756	1.1756
Price discrepancy					
Germany	0.9570	0.9569	0.9966	1.0017	1.0000
Ireland	0.9500	0.9501	0.9873	0.9983	1.0000
United Kingdom*	1.0000	1.0000	1.0000	1.0000	1.0000

D.2 Alternative reconciliation of 1999–2002 growth ratios for Domestic final demand

	Time series based growth	Substitution of – ECP prices	ECP expenditures	Fisher Index	PPP-implied growth
Price					
Germany	1.0261	1.0689	1.0686	1.0740	1.0722
Ireland	1.1733	1.2200	1.2194	1.2330	1.2351
United Kingdom*	1.0623	1.0623	1.0623	1.0623	1.0623
Volume					
Germany	1.0126	0.9720	0.9745	0.9695	0.9712
Ireland	1.1802	1.1351	1.1243	1.1119	1.1100
United Kingdom*	1.1066	1.1066	1.1066	1.1066	1.1066
Expenditure					
Germany	1.0390	1.0390	1.0413	1.0413	1.0413
Ireland	1.3848	1.3848	1.3709	1.3709	1.3709
United Kingdom*	1.1756	1.1756	1.1756	1.1756	1.1756
Price discrepancy					
Germany	0.9570	0.9970	0.9966	1.0017	1.0000
Ireland	0.9500	0.9878	0.9873	0.9983	1.0000
United Kingdom*	1.0000	1.0000	1.0000	1.0000	1.0000

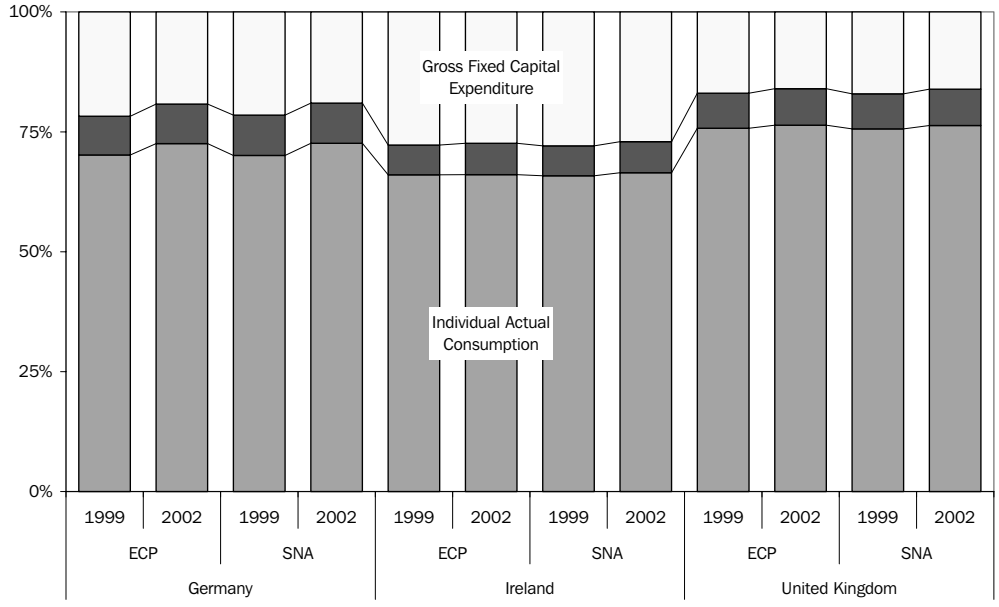
D.3 Reconciliation of 1999–2002 growth ratios for Domestic final demand – Price effects

	Time series based growth	Substitution of – Fisher Index	ECP expenditures	ECP prices IAC	CC	GFCE	PPP-implied growth
Price							
Germany	1.0261	1.0277	1.0278	1.0776	1.0807	1.0740	1.0722
Ireland	1.1733	1.1755	1.1754	1.2509	1.2527	1.2330	1.2351
United Kingdom*	1.0623	1.0623	1.0623	1.0623	1.0623	1.0623	1.0623
Volume							
Germany	1.0126	1.0110	1.0131	0.9663	0.9635	0.9695	0.9712
Ireland	1.1802	1.1780	1.1664	1.0959	1.0944	1.1119	1.1100
United Kingdom*	1.1066	1.1066	1.1066	1.1066	1.1066	1.1066	1.1066
Expenditure							
Germany	1.0390	1.0390	1.0413	1.0413	1.0413	1.0413	1.0413
Ireland	1.3848	1.3848	1.3709	1.3709	1.3709	1.3709	1.3709
United Kingdom*	1.1756	1.1756	1.1756	1.1756	1.1756	1.1756	1.1756
Price discrepancy							
Germany	0.9570	0.9585	0.9587	1.0051	1.0080	1.0017	1.0000
Ireland	0.9500	0.9518	0.9517	1.0129	1.0143	0.9983	1.0000
United Kingdom*	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

D.4 Reconciliation of 1999–2002 growth ratios for Domestic final demand – Expenditure effects

		Substitution of –					
	Time series based growth	*Fisher Index*	*ECP expenditures*				*PPP-implied growth*
			ECP prices	*IAC*	*CC*	*GFCE*	
Price							
Germany	1.0261	1.0277	1.0738	1.0737	1.0739	1.0740	1.0722
Ireland	1.1733	1.1755	1.2334	1.2330	1.2330	1.2330	1.2351
United Kingdom*	1.0623	1.0623	1.0623	1.0623	1.0623	1.0623	1.0623
Volume							
Germany	1.0126	1.0110	0.9675	0.9675	0.9694	0.9695	0.9712
Ireland	1.1802	1.1780	1.1227	1.1093	1.1092	1.1119	1.1100
United Kingdom*	1.1066	1.1066	1.1066	1.1066	1.1066	1.1066	1.1066
Expenditure							
Germany	1.0390	1.0390	1.0390	1.0388	1.0410	1.0413	1.0413
Ireland	1.3848	1.3848	1.3848	1.3677	1.3676	1.3709	1.3709
United Kingdom*	1.1756	1.1756	1.1756	1.1756	1.1756	1.1756	1.1756
Price discrepancy							
Germany	0.9570	0.9585	1.0016	1.0014	1.0016	1.0017	1.0000
Ireland	0.9500	0.9518	0.9987	0.9983	0.9983	0.9983	1.0000
United Kingdom*	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

D.5 Expenditure shares



E. THEORETICAL INCONSISTENCIES: FURTHER INSIGHTS

In Section 5, we found the index substitution effect to be rather enigmatic. The magnitude of the effect varied apparently inexplicably with the choice of reference country. Most significantly, in the case of Germany and the United Kingdom we observed no reciprocal relationship when we reversed the roles of subject and reference country (contrary to our observations for price and expenditure effects). This result is particularly puzzling because the conditions which lead to a significant effect in one case are almost exactly replicated (albeit in inverse form) in the other case, for which no discernible effect is found.

In this section, we do not completely resolve our confusion, but we do present a mathematical framework which we believe may be useful for further consideration of these issues.

We commence at the point in Section 5 where all ECP prices and expenditures have been substituted – table 5.1(c).

We represent component prices by

$$p_i^{s,y} \text{ and } p_i^{r,y}$$

where s denotes the subject country, r denotes the reference country, y is the year and i distinguishes the three sub-aggregates – *Individual actual consumption* (i=1), *Collective consumption* (i=2) and *Gross fixed capital expenditure* (i=3).

Component expenditures are similarly represented by

$$c_i^{s,y} \text{ and } c_i^{r,y}.$$

Aggregate prices and expenditures are

$$p_{DFD}^{s,y},\ p_{DFD}^{r,y},\ c_{DFD}^{s,y} \text{ and } c_{DFD}^{r,y}.$$

Recapping from Section 5, 1999 prices are the 1999 benchmark PPPs, inclusive of EKS adjustments. 2002 component price levels are derived by applying PPP-implied price movements to the 1999 PPPs. 2002 aggregate price levels are derived by extrapolating the 1999 benchmarks by Paasche indexes of temporal price change calculated from the component price and volume data.

We denote these temporal Paasche indexes by

$$K_{DFD}^{s} = \frac{p_{DFD}^{s,2002}}{p_{DFD}^{s,1999}} \text{ and } K_{DFD}^{r} = \frac{p_{DFD}^{r,2002}}{p_{DFD}^{r,1999}}.$$

If we define S^r_{DFD} to be the time series-based measure of aggregate price change in the reference country, we can define the PPP-implied measure of temporal price change at this point in the reconciliation process to be

$$\frac{K^s_{DFD}}{K^r_{DFD}} \times S^r_{DFD}.$$

In the final stage, we recompute the aggregate price measures for the subject country by extrapolating the corresponding reference country price measures by spatial Fisher indexes, as follows:

$$\text{(new)}\, p^{s,1999}_{DFD} = F^{1999}_{DFD} \times p^{r,1999}_{DFD} \quad \text{and} \quad \text{(new)}\, p^{s,2002}_{DFD} = F^{2002}_{DFD} \times p^{r,2002}_{DFD}.$$

The PPP-implied measure of temporal price change is now

$$\frac{F^{2002}_{DFD}}{F^{1999}_{DFD}} \times S^r_{DFD}.$$

The difference between this PPP-implied measure and the preceding measure defines the theoretical inconsistency we wish to examine.

We begin by separating the Fisher index formulae into Laspeyres and Paasche indexes,

$$\left[\frac{F^{2002}_{DFD}}{F^{1999}_{DFD}}\right]^2 = \frac{L^{2002}_{DFD} \cdot P^{2002}_{DFD}}{L^{1999}_{DFD} \cdot P^{1999}_{DFD}} = \frac{\sum w^{r,2002}_i R^{2002}_i \left[\sum w^{s,2002}_i \Big/ R^{2002}_i\right]^{-1}}{\sum w^{r,1999}_i R^{1999}_i \left[\sum w^{s,1999}_i \Big/ R^{1999}_i\right]^{-1}}$$

which we then define in terms of expenditure weights,

$$w^{s,y}_i = \frac{c^{s,y}_i}{c^{s,y}_{DFD}} \quad \text{and} \quad w^{r,y}_i = \frac{c^{r,y}_i}{c^{r,y}_{DFD}}$$

and spatial price relatives at the component level,

$$R^y_i = \frac{p^{s,y}_i}{p^{r,y}_i}.$$

We proceed by comparing the component price relatives with their corresponding aggregate price relatives:

$$R^{1999}_i = \frac{p^{s,1999}_i}{p^{r,1999}_i} = \lambda^{1999}_i \frac{p^{s,1999}_{DFD}}{p^{r,1999}_{DFD}} \quad \text{and} \quad R^{2002}_i = \frac{p^{s,2002}_i}{p^{r,2002}_i} = \lambda^{2002}_i \frac{p^{s,2002}_{DFD}}{p^{r,2002}_{DFD}}.$$

Substituting into the previous expression, we obtain

$$\left[\frac{F_{DFD}^{2002}}{F_{DFD}^{1999}}\right]^2 = \frac{\sum w_i^{r,2002}\lambda_i^{2002}\left[\sum w_i^{s,2002}/\lambda_i^{2002}\right]^{-1}}{\sum w_i^{r,1999}\lambda_i^{1999}\left[\sum w_i^{s,1999}/\lambda_i^{1999}\right]^{-1}} \times \left[\frac{p_{DFD}^{s,2002}\cdot p_{DFD}^{r,1999}}{p_{DFD}^{r,2002}\cdot p_{DFD}^{s,1999}}\right]^2$$

or, more succinctly,

$$\frac{F_{DFD}^{2002}}{F_{DFD}^{1999}} = \phi\frac{K_{DFD}^{s}}{K_{DFD}^{r}}$$

where

$$\phi = \sqrt{\frac{\sum w_i^{r,2002}\lambda_i^{2002}\left[\sum w_i^{s,2002}/\lambda_i^{2002}\right]^{-1}}{\sum w_i^{r,1999}\lambda_i^{1999}\left[\sum w_i^{s,1999}/\lambda_i^{1999}\right]^{-1}}}$$

directly measures the discrepancy introduced by the substitution of the Fisher index formulae. That is, we have defined the theoretical discrepancy as a function of expenditure weights (w_i) and scaling parameters (λ_i) which identify whether purchasing power parities are uniform across consumption and investment goods and services.

Our first observation is that, if $\lambda_i^{1999}=1,\forall i$ and $\lambda_j^{2002}=1,\forall j$, then $\phi\equiv 1$, and there is no theoretical discrepancy, since all expenditure weights sum to unity. Unfortunately, this is perhaps the only straightforward conclusion we can arrive at, as we shall demonstrate in a case study based upon table E.1.

E.1 Derivation of the theoretical inconsistency for the United Kingdom, using Germany as the reference country

	1999			*2002*		
	IAC	*CC*	*GFCE*	*IAC*	*CC*	*GFCE*
$w_i^{s,y}$	0.7573	0.0734	0.1693	0.7637	0.0763	0.1600
$w_i^{r,y}$	0.7017	0.0808	0.2175	0.7255	0.0824	0.1921
λ_i^y	1.0164	0.8466	1.0074	0.9925	0.8904	1.0895
L^y		1.0008			1.0027	
P^y		1.0002			0.9980	
ϕ			0.999881			

The source data for table E.1 can be found in tables 5.1(c) and 5.1(d), and the result has been reported previously in table 5.2 and figure 5.4 The indexes L^y and P^y in table E.1 are defined as follows:

$$L^y = \sum w_i^{r,y} \lambda_i^y \quad \text{and} \quad P^y = \left[\sum w_i^{s,y} / \lambda_i^y \right]^{-1}$$

allowing us to define

$$\phi = \sqrt{\frac{L^{2002}.P^{2002}}{L^{1999}.P^{1999}}}.$$

Note that L^y is a Laspeyres-type index employing the expenditure weights of the reference country and P^y is a Paasche-type index employing the expenditure weights of the subject country. In general, as we wish to make no *a priori* assumptions about spatial price comparisons in different benchmark years, we must consider the numerator and denominator in the above expression for ϕ independently. Hence, a general condition for the theoretical inconsistency to be small is that

$$\sqrt{L^{1999}.P^{1999}} \approx 1 \approx \sqrt{L^{2002}.P^{2002}}.$$

We observe that these conditions and the predicted outcome are satisfied by the data in table E.1. Note that both L^{1999} and P^{1999} are close to unity. While the spread between L^{2002} and P^{2002} is much wider, they are virtually reciprocals.

It is perhaps more informative to note other features of the data in table E.1. In expenditure terms, the structure of the subject country and reference country are quite similar but not overly so, and the expenditure weights for both countries change between 1999 and 2002. (See figure D.5 for a summary of expenditure weights.)

Most interesting is the fact that purchasing power parities at the sub-aggregate level are very heterogeneous in both years, and do not evolve in any uniform way between benchmark years. While some degree of heterogeneity is necessary for theoretical inconsistencies to exist, obviously the presence of heterogeneous characteristics *per se* does not ensure theoretical inconsistencies.

In table E.2 we look at the theoretical inconsistency for Germany, using the United Kingdom as the reference country. The expenditure weights in table E.2 are identical to those in table E.1, except that the subject and reference country roles are reversed. The λ_i for 1999 are the inverses of the λ_i reported in table E.1, and the L^{1999} and P^{1999} indexes also have an inverse relationship with the P^{1999} and L^{1999} indexes from table E.1. If the λ_i for 2002 were also the inverses of the λ_i reported in table E.1, then our calculation of ϕ would be the inverse of ϕ reported in table E.1, and similarly insignificantly different from unity. However they are not.

E.2 Derivation of the theoretical inconsistency for Germany, using the United Kingdom as the reference country

	1999			*2002*		
	IAC	*CC*	*GFCE*	*IAC*	*CC*	*GFCE*
$w_i^{s,y}$	0.7017	0.0808	0.2175	0.7255	0.0824	0.1921
$w_i^{r,y}$	0.7573	0.0734	0.1693	0.7637	0.0763	0.1600
λ_i^y	0.9838	1.1812	0.9927	**1.0125**	**1.1287**	**0.9224**
L^y		0.9998			1.0070	
P^y		0.9992			1.0022	
ϕ			1.005076			

The difficulty with understanding table E.2 is that the λ_i for 2002 are in fact very similar to the inverses of the corresponding factors in table E.1. It is really not until we calculate L^{2002} and P^{2002} that we realise that the theoretical inconsistency is much larger in this case. Hence we conclude that there may be only a very subtle difference between the conditions that lead to a significant inconsistency and those that do not.

For comparison, table E.3 gives details of the theoretical discrepancy for Ireland, using the United Kingdom as reference country. This is the largest discrepancy identified in our case study: $\phi = 1.0112$.

E.3 Derivation of the theoretical inconsistency for Ireland, using the United Kingdom as the reference country

	1999			*2002*		
	IAC	*CC*	*GFCE*	*IAC*	*CC*	*GFCE*
$w_i^{s,y}$	0.6603	0.0622	0.2775	0.6608	0.0652	0.2741
$w_i^{r,y}$	0.7573	0.0734	0.1693	0.7637	0.0763	0.1600
λ_i^y	1.0199	1.0318	0.9331	1.0499	1.0102	0.8979
L^y		1.0061			1.0225	
P^y		0.9949			1.0009	
ϕ			1.011166			

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